Computational Hemodynamics of Abdominal Aortic Aneurysms

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An existing computational model is extended to simulate Newtonian and non-Newtonian blood flow in three dimensional idealized abdominal aortic aneurysm (AAA) geometries in order to calculate the wall shear stress (WSS) and wall shear stress gradient (WSSG) along the arterial wall and obtain the velocity profile. From these, the locations of high and low WSS and WSSG can be determined, as well as regions of recirculation that present as low velocity vortices within the aneurysm. Results of these simulations will be analyzed to determine the effect of fluid constitutive law on the hemodynamics of idealized AAA geometries. This analysis will allow for conclusions to be made concerning the importance of using of a non-Newtonian constitutive law for blood flow in models of relatively large arteries.

I. INTRODUCTION

The aorta is a 2.0 cm diameter artery that delivers oxygen-rich blood from the heart to the iliac arteries. Abdominal aortic aneurysms (AAAs) are defined as a ballooning of the aorta below the kidneys to a diameter of 3.0 cm or more, and in general this structure is referred to as an aneurysm bleb. These aneurysms are the 13th leading cause of death in the United States, and are prevalent in 1-2% of adults. In older age groups (60+ years of age), this prevalence can be as high as 10% of all adults. Rupture of previously undetected aneurysms present a mortality rate of about 90% [1, 2].

Existing research states that irregular AAA geometries lead to complicated flow dynamics including vortices and regions of both high and low blood wall shear stress (WSS) [4]. Additionally, in aneurysm blebs, the blood recirculates at a slow velocity and creates varied, swirling vortices. These regions of the arterial wall that are under high and low WSS can lead to vascular tissue degradation [3], and areas along aneurysm wall with a large wall shear stress gradient (WSSG) can lead to further growth and rupture [5]. Previous computational studies investigated steady flow through two dimensional unilateral and bilateral AAA geometries, and then time-dependent pulsatile flow through two dimensional irregular geometries with respect to the effects on blood WSS [4]. These simulations showed that high regions of WSS occur at the proximal and distal ends of the aneurysm where velocity is highest, while low regions of WSS occur along the bleb wall where velocity is lowest. Also, the WSSG increased proportionally to bleb diameter supporting the literature that states larger aneurysms are more likely to grow and rupture [3, 5]. In this proposed project, we seek to extend two-dimensional simulations to three spatial dimensions and to extend simulations of Newtonian flow to non-Newtonian flow. The result will be a computational model of non-Newtonian flow in three dimensional idealized aneurysm geometries.

Newtonian fluids are fluids that exhibit a linear relationship between the fluid stress and strain rate, modeled with a constant kinematic viscosity. Alternatively, non-Newtonian fluids are fluids with a complicated microstructure that induces flow behavior that differs from Newtonian fluids; most often, these fluids exhibit a viscosity that is dependent upon a shear rate or its history, often in a nonlinear fashion.

For non-Newtonian fluids, a constant viscosity cannot be defined and thus the majority can be classified as shear-thickening or shear-thinning. Shear-thickening means that as the shear rate is increased, the viscosity increases as well. A simple example of a shear-thickening fluid is oobleck (a cornstarch and water mixture) that is thin with no or very low velocity, but thickens considerably when accelerated. Conversely, shear-thinning means that as the shear rate is increased, the viscosity decreases. A simple example of a shear-thinning fluid can be found in most latex paints - off the shelf, the viscosity is large and the paint is difficult to stir. However, as the stirring continues, thereby increasing the fluid velocity and shear rate, the viscosity of the paint decreases and the paint becomes easier to stir.

At the most basic level, blood is a suspension of cells in plasma. In consideration of the primary function of the aorta, the cells found within the blood flowing through an AAA are red blood cells. At low shear stress rates, these red blood cells aggregate into large stacks known as rouleaux. Due to this physical structure and behavior, blood is generally regarded as a shear-thinning, non-Newtonian fluid that may also exhibit viscoelastic effects [6]. However, most of the current research that focuses on computational simulations of AAAs assumes the fluid (blood) is Newtonian. While this assumption is usually rationalized by pointing to the small-scale and low velocity behavior of blood flow in arteries, it is also due to limitations in available computational fluid dynamics software and numerical methods for complicated non-Newtonian constitutive laws. This assumption continues to be made even when there is ample evidence that the choice of constitutive law has a significant effect on important fluid measures such as WSS [7]. Thus, despite the scale of the aorta, modeling the blood flow with a non-Newtonian constitutive law is vital to properly understanding the hemodynamical behavior and computing accurate measures associated with the flow.

II. GOALS

The proposed project seeks to compute accurate simulations of both Newtonian and non-Newtonian fluid dynamics in idealized aneurysm-like geometries and discern whether the Newtonian assumption has a significant effect on important hemodynamic measures (such as WSS and WSSG) that can lead to the further growth and rupture of AAAs [3–5].

III. METHOD

We intend to extend our existing computational fluid dynamics code and idealized aneurysm model from two dimensions to three dimensions and from Newtonian to non-Newtonian flow. These extensions will allow for comparisons to be made between several choices of fluid constitutive laws. From this comparison, conclusions concerning the significance of non-Newtonian flow in computational hemodynamical models will be determined.

A. Navier Stokes Equations

The incompressible Navier Stokes equations describe the motion of viscous fluids. The variables and parameters in the equations are found in Table I. The Navier-Stokes equations model conservation laws of momentum and mass:

$$\begin{aligned} &\boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nabla\cdot\boldsymbol{\sigma} = \boldsymbol{f}, && \text{(momentum)} \\ &\nabla\cdot\boldsymbol{u} = 0, && \text{(mass)} \end{aligned}$$

together with appropriate boundary conditions. The Newtonian assumption of a linear stress-rate of strain relationship is modeled by the constitutive law

$$\boldsymbol{\sigma} = \nu\left(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T\right) - pI.$$

The fluid wall shear stress WSS is defined by

$$\boldsymbol{w} = \boldsymbol{\sigma}\boldsymbol{n} - (\boldsymbol{n}^T\boldsymbol{\sigma}\boldsymbol{n})\boldsymbol{n}.$$

$\boldsymbol{u}$	Fluid Velocity (vector)
p	Fluid Pressure (scalar)
$\boldsymbol{\sigma}$	Fluid Stress (tensor)
ν, ν_i	Viscosity parameters (scalar)
$\boldsymbol{n}$	Outward unit normal (vector)
$\boldsymbol{w}$	Wall Shear Stress (vector)

TABLE I. Variables and Parameters

Several non-Newtonian constitutive laws that model shear-thinning behavior will be investigated, including the Power law

$$\boldsymbol{\sigma} = \left(\nu_0 + \nu_1 |\boldsymbol{D}(\boldsymbol{u})|^{(q-2)/2}\right) \boldsymbol{D}(\boldsymbol{u}) - pI,$$

where $\boldsymbol{D}(\boldsymbol{u}) = (1/2)(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$ is the deformation tensor, and the Carreau law

$$\boldsymbol{\sigma} = \left(\nu_0 + \nu_1 \left(\nu_2 + |\boldsymbol{D}(\boldsymbol{u})|\right)^{(q-2)/2}\right) \boldsymbol{D}(\boldsymbol{u}) - pI.$$

When $0 < q < 2$ in the above models, these represent shear-thinning behavior. Note that if $q = 2$, then the Newtonian constitutive law (i.e., constant viscosity) is recovered.

The above models were selected because they provide the foundation of most constitutive models of blood rheology [8]. The power law gives a simple example of a way to model a generalized Newtonian fluid, and the form of the viscosity function allows for easy changes from Newtonian to shear thinning fluids [6]. The Carreau law draws on an analogy between polymer network theories and the aggregation of red blood cells in rouleaux networks at low shear stress rates to more accurately incorporate the physical behavior of blood into the model [6].

B. Computational Methods

The existing computational approach implements a finite element method to solve the Navier-Stokes equations in the aneurysm geometry. Dirichlet boundary conditions are are used to specify the inflow and outflow velocities. The aneurysm geometry itself is formed from cubic spline interpolations, and then a mesh generation tool is used to create a Delaunay triangulation of the spatial domain from the boundary specifications. An additional step computes the barycentric refinement of each mesh. Within the finite element method, the backward Euler method is used to step forward in time, and a Newton iteration is used to solve the system of nonlinear equations at each time step. Once the fluid velocity and pressure are computed, they are then used to calculate the blood wall shear stress and its gradient. Overall, this model is formed and simulations are conducted in the open source finite element software FreeFEM++. The results of these simulations are extracted and then analyzed with MATLAB and VisIt visualization software. These steps allow for the visualization of the blood velocity, WSS, and WSSG. These profiles will reveal the impact of the fluid constitutive model on the hemodynamic parameters of WSS and WSSG.

IV. RESOURCES

For this project, research facilities within the Department of Mathematics will be used. The only instrument needed is a College of Charleston research computer, along with some open source fluid flow software. These resources are already on hand, and no additional resources are requested or needed.

V. BUDGET

No funds will be necessary for the completion of this project.

Objective	Target Date (2015/2016)
Implement 3D mesh and rough draft of paper with past results	Aug. 1
Data collection, analysis, and comparison to 2D	Sept. 1
Implement non-Newtonian flow	Oct. 1
Data collection, analysis, and comparison to Newtonian Flow	Nov. 1
Updated draft of paper	Dec. 1
Explore new geometries (possibly patient-specific)	Jan. 1
Data collection and analysis	Feb. 1
Final draft of paper	March 1
Final poster finalized	April 1
SSM Poster Presentation	Mid-April
Departmental Talk	Late April

TABLE II. Timeline of Project Tasks

VI. TIMELINE

Work for this project will be conducted over the course of the 2015-2016 academic year in order to complete a Bachelor's Essay as given in Table II.

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