Developing a Z-R Relationship with Uniform Sampling

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ABSTRACT

A new method for developing a relationship relating the rain rate, R, to the radar reflectivity factor, Z, will be developed to reduce errors from the conventional method. A two-dimensional video disdrometer will be used to separate rain events into uniform samples containing the same number of drops, as opposed to the conventional temporal based samples containing varying number of drops. Z-R relationships will then be developed using the samples of uniform drop number, reducing error associated with the idiosyncrasies of samples derived from uniform sampling time. The new method is expected to result in Z-R relationships of increased accuracy compared to those derived from the conventional non-uniform temporal sampling.

1. Introduction

The relationship between radar, specifically a quantity called the radar reflectivity factor, Z, and the rain rate, R, has been a topic of discussion and debate in the atmospheric physics community since the middle of the 20th century when it was first proposed by Marshall et al. (1947). The radar reflectivity factor is a quantity derived from the power return of radar by the weather-radar equation, Battan (1973),

$$P_r = C\frac{|K|^2}{r^2}Z, \tag{1}$$

where P_r is the power return of the radar, K is a constant related to the dietetic properties of water, C is a constant related to the electronic gains of the radar, r is the distance of the detected drops from the radar source, and Z is the radar reflectivity factor. Both Z and R can be written as moments of the drop size distribution, or the probability distribution of the drop sizes in a rain event. Marshall and Palmer (1948) modeled the drop size distribution as a function of the drop size,

$$N(D) = N_0 e^{-\Lambda D}, \tag{2}$$

where N_0 depends on the storm system and $\Lambda = 4.1R^{-0.21}$, an empirically determined quantity. In actuality the drop size distribution varies significantly in and between rain events; thus this assumption is inaccurate and is a source of error in Z-R relationships. However, in the event of large number of drops the error of this assumption is decreased, thus it is often used to model rain events.

Z can be written as the sixth moment of the drop size distribution,

$$Z = \frac{1}{V_s}\sum_{i=1}^{N} D_i^6 \tag{3}$$

where V_s is the sample size and D_i is the drop diameter of the ith drop within the distribution of drop sizes. R can similarly be written,

$$R = \frac{1}{V_s}\sum_{i=1}^{N} \frac{\pi}{6} D_i^3 V_t(D_i), \tag{4}$$

where V_t is the terminal velocity of drops of diameter D_i. A relationship between Z and R is physically feasible due to their mutual dependence on drop diameter and sample size, seen in Equations 3 and 4. As first shown by Marshall et al. (1947), a relationship between these two quantities is developed empirically and typically takes the form of a power law,

$$Z = aR^b, \tag{5}$$

where a and b are fit parameters determined either empirically through fitting experimental data or analytically by exploring different forms of the raindrop size distribution. The use of a power law in this relationship stems from the representation of Z and R in logarithmic scales due to their wide range of possible values; applying a linear fit on a log-log plot most naturally takes the form of a power law. Several hundred specific Z and R relationships with different a and b values depending on factors such as storm type, season, geographical location, etc, have been published since the relationship was first verified in 1947, e.g. Battan (1973).

There are several issues with Z-R relationships leading to large discrepancies between the rain rate inferred by radar and the rain rate measured on the ground. One of these issues is due to sampling variability in the formulation of Z-R relationships, e.g.Larsen (2006). Samples are taken on a temporal basis, every five minutes for certain radar systems or every minute for optical disdrometers. Because of this temporal based sampling, the samples do not contain the same number of drops and are thus, in a sense, non-uniform. Two samples from the same storm could have great variance in drop number, yet still hold the same weight in determining Z-R relationships. This source of error is further increased due to the assumption of a uniform drop size distribution in storm events (Equation 2) which produces an increased error in cases of samples with low drop number.

As first discussed by Larsen (2006), this leads to the need for a new look at Z-R relationships, beyond resolving them for specific cases by producing new fit parameters. The relationship, as it is currently derived, has core flaws that could not be resolved simply by continually formulating new relationships out of an old method.

2. Goals

For this project I aim to develop a Z-R relationship using samples determined by number of drops, rather than on a temporal basis as has been historically done. This will produce a uniform sampling base, with each sample containing the same number of drops, for the empirical determination of Z-R relationships. Such a shift in the development of the relationship is expected to lead to a more accurate relationship and therefore greater accuracy in estimating rain rate using radar.

3. Method

To develop a Z-R relationship using uniform sampling, a two-dimensional video disdrometer (2DVD) will be used in deriving samples containing the same number of drops. The 2DVD records the number of drops, individual drop size, velocity, and several other quantities with microsecond resolution during rain events. This data, which has already been acquired, will allow the creation of samples containing the same number of drops.

A code will be written in MATLAB to divide each selected rain event into multiple samples each containing the same number of drops. The code will count each entry in the data, representing each individual drop, up to a specified number and create a sample. The code will repeat the process until the end of the rain event. For comparison, a temporal based sampling of each event will also be produced within MATLAB where a separate code will divide each rain event into multiple samples of the same temporal length.

From these samples, R can be determined simply as the depth of rain accumulating per time by,

$$R = \sum_{i=1}^{N} \frac{\pi D_i^3}{6 A_s t}, \qquad (6)$$

where A_s is the sample area of the two dimensional video disdrometer and t is the time period of each segment. Z can be determined using Equation 3. Due to the use of a two-dimensional video disdrometer to determine these relationships, adjustments must be made for the sample volume, V_s, in the calculation of Z by Equation 3.

Z and R values will be found for several rain events of both convective and stratiform structure using both sampling of uniform drop number and temporal based sampling. Z-R relationships can be found through use of a MATLAB code to plot corresponding Z and R values from each sample in log-log space and find best-fit lines using a least-squares fit. The two types of relationships, based on temporal sampling and sampling of uniform drop number, will then be compared to determine if in fact uniform sampling increases the accuracy of Z-R relationships. Accuracy of the relationships can be assessed by using the relationship and 2DVD inferred Z values to calculate R values. If the relationship is 100% accurate, this process will return the same R values measured by the 2DVD. Thus the more accurate relationship will return a higher number of R values closer to the R value measured by the 2DVD.

4. Resources

This project requires the use of a two-dimensional video disdrometer which is already deployed and in use within a dense optical disdrometer array at College of Charleston's Dixie Plantation. Machines to store the data recorded by the instruments at Dixie Plantation are already in place. For coding and data analysis, computers equipped with MATLAB are also needed, and are currently in place in the Department of Physics and Astronomy at the College of Charleston. Ample data for this project has already been acquired and no additional budget nor resources are required.

5. Timeline

An expected timeline for this project including steps listed in the above method and dissemination goals can be found in Table 1.

REFERENCES

Battan, L., 1973: *Radar Observatin of the Atmosphere*. The University of Chicago Press, Chicago.

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Marshall, J. S., R. C. Langille, and W. M. Palmer, 1947: Measurement of rainfall by radar. *Journal of Meteorology*, **4**, 186–192.

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List of Tables

Table 1. Expected Timeline of Events for Project Spanning September 2015 - April 2016

Month	Expected Accomplishments
September	select rain events from previously recorded 2DVD data
	begin writing MATLAB codes to create temporal samples and create samples of uniform drop number
October	finish writing MATLAB codes to create samples
	begin running code on selected events
November	all rain events separated into samples by MATLAB code
December	write code to plot and fit Z-R data
January	process data using code and finish developing Z-R relationships
February	comparative analysis to determine accuracy
	begin writing paper
	South Carolina Academy of Science abstract deadline
March	finish writing paper
	begin prepping presentations in group research meetings
April	finalize paper editing and submit to journal
	presentations at
	South Carolina Academy of Science Annual Meeting
	School of Science and Math poster session
	Department of Physics and Astronomy Colloquium