Assignment I, PHYS 301 (Classical Mechanics)
Spring 2017
Due 1/27/17 at start of class

PLEASE READ THE COVER PAGE CAREFULLY – IT OUTLINES SOME BASIC PROTOCOLS FOR SOLVING HOMEWORK PROBLEMS IN THIS CLASS AND WON’T BE MENTIONED AGAIN!!!

In this class, you will be receiving weekly homework assignments. They are meant to be challenging, and likely will take a good chunk of your time. I apologize for this – but there’s really no way around it. One learns Physics by actually solving problems. If you “get the ideas” but can’t solve a problem associated with a particular concept, you have not yet mastered the concepts and tools you need to effectively “do” Physics. Everything always makes more sense when your professor is doing it on the board than when you are trying to do it yourself. For most, the best way to get better at solving problems is to get more practice. That’s why we have the homework. (Trust me; I don’t give this to you for my benefit.)

This course will be challenging, and this first homework is to ensure that you’re really prepared for this course. Thus, the first assignment is comprised of problems I asked my PHYS 111 class last semester on exams. Not all of these problems are hard. Mostly, this homework set is designed to be a review and a heads’-up to alert you to PHYS 111 topics you might need some extra review on.

Some other homework preliminaries that we need to get out of the way:

• **IN THIS COURSE, DO NOT USE MATHEMATICA, OTHER COMPUTATIONAL TOOLS, OR CALCULATORS UNLESS I SPECIFICALLY TELL YOU THAT YOU SHOULD!**

  • Successful students historically have started working on the homework right after turning in their previous homework assignments; if you wait until the night before an assignment is due to start it, you will almost certainly struggle. Think of attacking this homework as a daily ritual – if you work on it a bit each day, it isn’t nearly so daunting.

  • I do not require you to solve problems any particular way – however, a quickly drawn sketch often will help in mechanics problems. I suggest you might want to try that to just picture what is going on before starting to push around symbols.

  • You do not have to type your solutions – but it is appreciated if you do. I do expect that any solutions you give me are **legible** and easy to follow. When the problem asks you to find an expression, please **circle or box** the final answer so it is easier for me to find. Although it is not required, I do recommend you put each answer on a separate piece of paper. (My answer keys often will not put each answer on a spare sheet of paper, but that’s to save a bunch of paper – I make 20+ copies of each answer key, so saving half a page here and there really adds up in my case.)

  • Note – those of you who took other courses from me already probably realize this. Units are your friends. Checking the units of your final answer can help you see if you’ve made a mistake. (It might not tell you where you’ve made a mistake, but if your answer does not have the right units at the end, there must be a mix-up somewhere.) I use this trick all the time – and it can save you some serious anguish.

  • **YOU MUST LEAVE ANSWERS IN TERMS OF THE VARIABLES GIVEN IN THE PROBLEM STATEMENT ONLY.** If the problem refers to variables $m$, $M$, and $a$, only refer to $m$, $M$, and $a$ in your solution. (You may, however, also include constants like $2$, $\pi$, $\sqrt{2}$, $g$, $G$, etc.) I try to make the questions as clear as possible but, if you are in doubt, ask!
• More than anything else, please make your work clear and – especially if you aren’t 100% sure you are correct – EXPLAIN YOUR THOUGHT PROCESS!!! I can’t give you partial credit if I can’t figure out what you did. PLEASE DO NOT JUST GIVE ME A LONG STRING OF EQUATIONS AS AN ANSWER! This isn’t a Math class – we’re talking about physical systems. An equation doesn’t just come from nowhere. Give me context. Give me thoughts. Give me what ideas you are trying to use! You may earn a little bit more credit if you include this and – more importantly – if you give me TEXT in your answers, I can better help you figure out if you make a conceptual error. If all I have to grade are a bunch of equations right after each other in a row, it is difficult for me (or anyone else) to figure out what you were doing. I will always be happier if you include more words to describe your ideas/reasoning/thoughts. I know it takes a bit more work, but the more you show me about your lines of thought, the more effective I can be in helping you attack these problems. You don’t necessarily have to write in complete sentences, but sometimes a word or two makes all the difference in helping me understand what you were trying to do. (Of course, you don’t have to take this to extremes – if you divide both sides of an equation by 2, I can probably figure that out without any text..... when in doubt, include more words. It can’t hurt.)

• You may work with your classmates on the homework, but it is expected that your final solutions are your own. Collaborate/discuss/work out elements of the problem with each other if you wish – but when it comes time to write your actual solution, don’t copy from each other – get the basic ideas of what you have to do and then go off and write your own answers. Since organization, legibility, and text counts – don’t be surprised if you work with someone, get the same answer, but a different grade! I’m grading the whole solution here – not just the answer.

• Finally, I hope you are able to find this class enjoyable – despite the fact that it is a lot of work. Learning this stuff is critical to your development as a scientist, and I will do absolutely everything I can to help you succeed. Working hard in this class will likely pay serious dividends down the road.

We’ll start you out with some basic multiple choice. You need not show any work for these; they will be graded merely on whether or not you got them right. (These will likely be the only multiple-choice questions you are asked to answer all semester).

1. A person standing at the edge of a cliff throws a ball straight up and another ball straight down with the same initial speed. Neglecting air resistance, the ball to hit the ground below the cliff with the greater speed is the one initially thrown:
   (a) upward
   (b) downward
   (c) neither – they both hit with the same speed

2. You are throwing a ball straight up in the air. At the highest point, the ball’s:
   (a) velocity and acceleration are zero
   (b) velocity is nonzero but its acceleration is zero
   (c) acceleration is nonzero, but its velocity is zero
   (d) velocity and acceleration are both nonzero
3. You are pushing a wooden crate across the floor at constant speed. You decide to turn the crate on end, reducing by half the surface area in contact with the floor. In the new orientation, to push the same crate across the same floor with the same speed, the force that you apply must be:
(a) four times as great
(b) twice as great
(c) equally as great
(d) half as great
(e) one-fourth as great
as the force you required before you changed the crate’s orientation.

4. A spring of force constant $k$ is stretched a certain distance. It takes twice as much work to stretch a second spring by half this distance. The force constant of the second spring is:
(a) $k$
(b) $2k$
(c) $4k$
(d) $8k$
(e) $16k$

5. A spring-loaded toy dart gun is used to shoot a dart straight up in the air, and the dart reaches a maximum height of 24 m. The same dart is shot straight up a second time from the same gun, but this time the spring is compressed only half as far before firing. How far does the dart go this time, neglecting friction and assuming an ideal spring?
(a) 96 m
(b) 48 m
(c) 24 m
(d) 12 m
(e) 6 m
(f) 3 m
(g) Impossible to determine

6. Suppose you are on a cart, initially at rest on a track with negligible friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure below, is the cart put into motion?
(a) Yes, it moves to the right
(b) Yes, it moves to the left
(c) No, it remains in place
7. If all three collisions in the figure below are totally inelastic, which cause(s) the most damage? The most damage occurs when the greatest amount of kinetic energy is lost. (Only select an answer with two scenarios if there is a tie for the most loss in Kinetic Energy.) (The wall doesn’t move after colliding with the car).

(a) I
(b) II
(c) III
(d) I and II
(e) I and III
(f) II and III
(g) I, II, and III

8. A block of mass $m$ slides down an incline with initial speed $v$, starting at initial height $h$ above the ground, as shown in the figure below. The coefficient of kinetic friction between the mass and the incline is $\mu$. If the mass continues to slide down the incline at a constant speed, how much energy is dissipated by friction by the time the mass reaches the bottom of the incline?

(a) $mgh/\mu$
(b) $mgh$
(c) $\mu mgh/\sin \theta$
(d) $mgh \sin \theta$
(e) 0
9. A block is attached to a horizontal spring. On top of this block rests another block. The two-block system slides back and forth in simple harmonic motion on a frictionless horizontal surface. At one extreme end of the oscillation cycle (when the blocks come to a momentary halt before reversing the direction of their motion), the top block is suddenly lifted vertically upward, without disrupting the lower block in any way. The simple harmonic motion of the bottom block then continues. What happens to the amplitude of the motion block that remains in simple harmonic motion?

(a) The amplitude increases
(b) The amplitude decreases
(c) The amplitude remains the same
(d) It is impossible to tell

10. A block is attached to a horizontal spring. On top of this block rests another block. The two-block system slides back and forth in simple harmonic motion on a frictionless horizontal surface. At one extreme end of the oscillation cycle (when the blocks come to a momentary halt before reversing the direction of their motion), the top block is suddenly lifted vertically upward, without disrupting the lower block in any way. The simple harmonic motion of the bottom block then continues. What happens to the oscillation frequency of the block that remains in simple harmonic motion?

(a) The oscillation frequency increases
(b) The oscillation frequency decreases
(c) The oscillation frequency remains the same
(d) It is impossible to tell

11. A block is attached to a horizontal spring. On top of this block rests another block. The two-block system slides back and forth in simple harmonic motion on a frictionless horizontal surface. At one extreme end of the oscillation cycle (when the blocks come to a momentary halt before reversing the direction of their motion), the top block is suddenly lifted vertically upward, without disrupting the lower block in any way. The simple harmonic motion of the bottom block then continues. What happens to the total mechanical energy of the system after the block is removed?

(a) The total mechanical energy of the system is increased by removing the block.
(b) The total mechanical energy of the system is decreased by removing the block.
(c) The total mechanical energy of the system remains unchanged by removing the block.
(d) It is impossible to tell

12. A hollow hoop, a uniform solid disk, and a uniform solid sphere – each of mass $M$ and radius $R$ – are released from rest down the same inclined wedge. They all roll down the wedge without slipping. Which choice correctly predicts the final order of arrival at the bottom of the inclined plane (from first arrival to final arrival)?

(a) hoop, disk, sphere
(b) hoop, sphere, disk
(c) disk, hoop, sphere
(d) disk, sphere, hoop
(e) sphere, hoop, disk
(f) sphere, disk, hoop
(g) all arrive simultaneously
(h) impossible to tell
Quantitative Problem Solving

Although most homework assignments this semester will involve symbolic answers – rather than numerical answers – these warm-ups will often involve numbers. For the rest of this homework assignment (only!), you may use calculators on problems that involve numerical answers! I will make it clear in each problem at the end if your answer should be numerical or symbolic by including (numerical) or (symbolic) parenthetically. Note that numerical answers should have units. (Symbolic answers have understood units).

13. A book of mass $M$ is positioned against a vertical wall. The coefficient of friction between the wall and the book is $\mu$. (We’ll set the coefficient of static friction and the coefficient of kinetic friction to be the same, for simplicity). You wish to keep the book from falling by pushing on it with a force $F$ applied at an angle $\theta$ with respect to the horizontal as shown below. $\theta$ is automatically constrained between $-\pi/2$ radians ($-90^\circ$) and $\pi/2$ radians ($+90^\circ$) from the geometry of the problem. Leave your answers to parts (b) and (c) in terms of $g$, $M$, $\mu$, and $\theta$.

(a) Draw a clear, well labeled free-body diagram for the book.

(b) For a given $\theta$, what is the minimum $F$ required to keep the book from falling? (symbolic)

(c) Is there a value of $\theta$, below which there does not exist an $F$ that keeps the book up? If so, find it. If not, briefly explain why not. (symbolic/explanation)

14. Tom, an engineering student, is going to take an exam but has forgotten his calculator. He stops outside his dorm and calls for help. Charlie, another engineering student 7.00 m above him on the third floor, drops a calculator (from rest) out the window.

(a) How much time is the calculator in the air? (numerical)

(b) What is the speed of the calculator just prior to being caught by Tom? (numerical)

(c) Danielle, a Physics student on the fourth floor 10.5 m above Tom, hears Tom’s call for help and throws a water balloon at him the same instant Charlie drops the calculator. What must be the initial speed of the water balloon if it reaches Tom at the same time as the calculator? (numerical)

15. A ball is thrown from the top of a tower that is 23 meters tall. The ball is thrown with initial speed 50 feet per second at an angle $\theta = +41^\circ$ with respect to the horizontal.

(a) How far from the base of the tower (in meters) does the ball land? (numerical)

(b) On a second throw, the thrower gets a ball to land in the same place as the previous problem, but it stays airborne one second longer. Is this second throw made with a greater or lesser launch angle? (answer greater or lesser)
16. Examine the figure below. A mass \(2M\) sits on a level platform, and is connected to masses \(3M\) and \(4M\) via massless inextensible strings as shown. The pulleys are frictionless (no rotation!), and this setup exists near the surface of the Earth.

(a) If the platform is frictionless, all of the masses have the same acceleration. Find this acceleration (in terms of \(g\)). (symbolic)

(b) Find the tension in each rope if the platform is frictionless. (symbolic)

(c) If none of the masses move, it must be because the platform has some friction. You observe the system, and none of the masses move. Find the smallest value that \(\mu_s\) could possible be in this case (in terms of \(M\) and \(g\)). (symbolic)

![Diagram of masses connected by strings]

17. An object with mass \(m\) and moving at 1-dimensional velocity \(+v\) relative to an observer explodes into 3 pieces, with masses in the ratio 1:3:6. The explosion takes place in deep space (no gravity to worry about). The least massive piece now moves at velocity \(-v\) (relative to the observer), and the second smallest mass now moves at velocity \(+2v/3\) (relative to the observer).

(a) How fast is the most massive piece moving (relative to the observer, in terms of \(v\))? (symbolic)

(b) What is the ratio of the final kinetic energy (after the explosion) divided by the initial kinetic energy (before the explosion) for the system (measured by the observer)? (numerical) (Leave your answer as a fully reduced/simplified fraction).

18. Two small spheres of putty, \(A\) and \(B\) of mass \(M\) and \(3M\), respectively, hang from the ceiling on strings of equal length \(\ell\). Sphere \(A\) is drawn aside so that it is raised to a height \(h_o\) as shown below and then released. Sphere \(A\) collides with sphere \(B\) at which point they stick together and (while attached to each other) swing to a maximum height \(h\), when the two spheres are momentarily at rest. What is \(h\) in terms of \(h_o\)? (symbolic)

![Diagram of spheres swinging]

19. Two astronauts, each with mass \(M\), are connected to each other by a rope of length \(d\) having negligible mass. They are isolated in space, orbiting their center of mass with each moving at speed \(v\). Calculate:

(a) The moment of inertia of the system (in terms of \(M\) and \(d\)). (Assume the astronauts are point particles). (symbolic)

(b) The magnitude of the angular momentum of the system with respect to the center of mass (in terms of \(M\), \(v\), and \(d\)). (Assume the astronauts are point particles). (symbolic)

(c) The rotational energy of the system (in terms of \(M\), \(v\) and \(d\)). (symbolic)
20. Neptune is a planet that is approximately spherical. Its mass is approximately $1.02 \times 10^{26}$ kg and its radius is about $2.46 \times 10^7$ m.

(a) What is the gravitational acceleration near the surface of Neptune? (in other words, what is $g_{\text{Neptune}}$)? (numerical)

(b) The escape velocity of a planet is how fast something must be moving away from the planet in order to break completely free of a planet’s gravitational pull. (In other words, it is moving with enough kinetic energy to eventually get an infinite distance away). You can find an escape velocity a number of different ways – one of the easiest to is to calculate the speed an object would be moving when it hits the surface of the planet if “dropped” from infinite distance away. What is the escape velocity for Neptune? (numerical)

(c) If you put a pendulum clock on Neptune and you want to design it so that it undergoes a full oscillation once every 7 seconds, how long should you make the arm of the pendulum? (Assume that the pendulum is a simple pendulum, with all the mass at the end, and the system undergoes small oscillations). (numerical)

21. Below is a picture of an Atwood Machine. Assume the (massless) rope moves on the pulley without slipping. $m_1 = 3$ kg, $m_2 = 2$ kg, the radius of the pulley $R = 5$ cm and the moment of inertia of the pulley is $0.01$ kg m$^2$. The system starts from rest at $t = 0$.

(a) What is the acceleration of $m_1$? (numerical).

(b) After 3 seconds, how many full rotations has the pulley gone through? (numerical).

22. Below is a picture of a uniform disk of mass $M = 5.00$ kg and radius $R = 0.35$ m that rolls (without slipping) down an incline of height $H = 3.00$ m with elevation angle $\theta = 20^\circ$. Embedded within the disk at the edge are four point masses as shown. These point masses each have mass $m = 1.25$ kg (so the total mass of the disk with the embedded masses is 10.00 kg). What is the speed of the disk upon reaching the bottom of the incline? (numerical)

23. A mass of $M = 2.8$ kg slides on a frictionless tabletop and is connected to a horizontal spring of $k = 320$ N/m. The mass is displaced from equilibrium by a distance of 3.5 cm and then released (from rest) at time $t = 0$. What is the speed of the mass at $t = 2.3$ seconds? (numerical)
24. A baseball ($m = 0.145$ kg) is hit at an initial speed of 39 m/s at $20^\circ$ above the horizontal. A defensive player stands 45 meters away from the hitter and is directly in the path of the ball (in other words, the ball will go right over the defensive player’s head). Neglect any air resistance.
   
   (a) How long does it take for the baseball to be straight above the defensive player? (numerical)
   
   (b) What is the height of the baseball (measured above bat-level) when it reaches the defensive player? (numerical)
   
   (c) Your answer to the question above should reveal that the ball will easily go over the head of the defender. In desperation, the fielder takes off his glove (mass = 0.555 kg) and throws it straight up in the air 0.5 seconds after the ball hit the bat. What must be the speed at which the glove is thrown so that it hits the ball at the exact correct time? (For simplicity, assume that the glove was thrown from the same height that the ball left the bat). (numerical)
   
   (d) In the scenario described above, assume that the glove catches the ball perfectly in an inelastic collision. You know (from your answers to the previous parts of this question) the distance from the batter and the height at which this collision occurs. What is the velocity (not just speed!) of the glove-ball system immediately after the collision? (numerical)
   
   (e) How much kinetic energy is lost in the collision process? [In other words, if you calculate the total kinetic energy of the glove + ball right before the collision and then calculate the kinetic energy of the glove/ball combo right after the catch, there will be a loss of some amount of kinetic energy. Calculate it). (numerical)

25. A lollipop is comprised of a thin stick of length $L = 10$ cm and mass $m = 5$ g with a spherical ball of radius $R = 3$ cm and mass $M = 45$ g as shown. The non-tasty end of the lollipop is fixed to a table with some chewing gum and is initially configured upright (as shown on the left side of the figure). At some point, a minor passing breeze disturbs the lollipop slightly and it begins to fall, leaving its end fixed in the chewing gum (see right side of the figure).
   
   (a) What is the moment of inertia of this lollipop with respect to the given pivot point? (numerical)
   
   (b) What is the kinetic energy of the lollipop at the moment before it hits the ground? [For simplicity, assume the lollipop hits the ground at $\theta = 0$, even though it technically would hit before that]. (numerical)
   
   (c) What is the angular velocity of the lollipop (aka $\frac{d\theta}{dt}$) at the moment just before the lollipop hits the ground? [Again, take “hitting the ground” to be $\theta = 0$]. (numerical)

26. (Extra Credit points) Return to problem 25. If you look carefully, you should realize that $\theta = 0$ is not really a realistic condition for the lollipop hitting the ground. Rather, the lollipop will hit the ground when the outer edge of the sphere hits the ground, which occurs at some $\theta > 0$. Let’s use the same basic setup as designed in problem 24, but specifically account for the fact that the lollipop hits the ground at some nonzero $\theta$. Let’s assume the lollipop starts upright at rest. Upon hitting the ground, we will say that 70% of the kinetic energy of the lollipop is lost to vibrations/noise/deformations and only the remaining 30% of the kinetic energy remains in the system. To what maximum angle $\theta'$ does the lollipop bounce to? Be careful! This isn’t easy! Also, since it is extra credit, I’m going to be a bit stingy with partial credit here. (numerical).