Recall that the definition of “phase velocity” $v_p = \frac{\omega}{k}$ and that the definition of “group velocity” is $\frac{d\omega}{dk}$.

1. Show that $v_g = v_p + k \frac{dv_p}{dk}$

2. In order to locate a particle to within $5 \times 10^{-12}$ meters using light, the wavelength of the light must be at most $5 \times 10^{-12}$ meters.
   a) Calculate the energy of a photon with $\lambda = 5 \times 10^{-12}$ m.
   b) Calculate the momentum of a photon with $\lambda = 5 \times 10^{-12}$ m.
   c) If this light bounces off an electron leaving an uncertainty $\Delta x = 5 \times 10^{-12}$ m to its position, what is the minimum uncertainty in the electron’s momentum?

3. An excited state of a certain nucleus has a half-life of 2.3 ns.
   a) Taking this to be the uncertainty $\Delta t$ for emission of a photon, calculate the minimum uncertainty in the frequency of the emitted light.
   b) If the emitted light is expected to have a wavelength 0.05 nm, what is $\Delta f/f$ for this light? ($\Delta f/f$ can be interpreted as the fractional uncertainty of the frequency).

4. Wave functions must be “normalized”. In other words, the integral:
   \[ \int_{-\infty}^{\infty} \Psi^*\Psi dx = 1 \]
   for a 1-dimensional system. Recall that $\Psi^*$ indicates the complex conjugate and, if $\Psi$ doesn’t have any complex quantities (no imaginary numbers), then $\Psi^* = \Psi$. Let $\Psi(x,0) = A|x|e^{-k|x|}$, with $A$ and $k$ unspecified constants. Find what $A$ has to be in terms of $k$ so that the wave function is properly “normalized”. (You may use on-line resources, integral tables, and/or computational tools to do the necessary integral for you. Be careful with the absolute values, though).

5. If we know that the integral $\int_{-\infty}^{\infty} \Psi^*\Psi dx = 1$, what do the units of $\Psi$ have to be?

6. The wave function describing a state of an electron confined to move along the $x$ axis is given at time zero by:
   \[ \Psi(x,0) = Ae^{-x^2/(4\sigma^2)} \]
   a) Where is the electron most likely to be found?
   b) What is the probability of finding the electron in a region $dx$ centered at $x = 0$?
   c) What is the probability of finding the electron in a region $dx$ centered at $x = \sigma$?
   d) What is the probability of finding the electron in a region $dx$ centered at $x = 2\sigma$?