Methods of Applied Physics
Course Content / Related Topics
(Approximate)

Week 1 Review of assorted concepts from precalculus and introductory calculus classes with an emphasis on skills regularly needed in upper-level Physics coursework. Includes basic graphing (to emphasize limiting cases in expressions as well as critical/inflection points, asymptotic behavior, etc.) Refresher on series expansions for e^x, $\ln(x)$, $\sin(x)$, $\cos(x)$, $(1+x)^n$ (especially $n = \pm\frac{1}{2}$), etc. Taylor/Maclaurin series approximations. Leading term approximation. Large or small parameter substitution for numerical approximations to integrals, etc. Refresher on summing geometric series.

- *Physics context - extremely broad. Examples from all-over, e.g. Maxwell-Boltmann Velocity Distribution Function, Gamma probability density function, Hydrogenic Radial Wavefunction solutions, Bose-Einstein Occupation Statistics, Planck Blackbody Equation, etc.) Relativistic corrections in the small v/c limit, period of slightly damped oscillators, quick numerical approximation of roots, Stirling's formula / Statistical mechanics, connection between geometric series and exponential behavior.*

Week 2 Basic introduction to ideas behind complex variables. Transformation between polar and Cartesian forms. Notion of conjugate and norm. Nth roots of 1. Log as a multivalued function. Quaternions.

- *Physics Context - Familiarity with Euler identity. Different ways of describing a plane wave. Relationships between hyperbolic trig functions and standard trig functions. Impedance of capacitors and inductors. Mueller matrices. Description of Phase. Description of rotations in 2d. Solution to damped/driven simple harmonic oscillator. Use in Schrödinger equation. Derivation of velocity/acceleration in polar coordinates. Conceptual introduction to residue theory.*

Week 3 Introduction/Review of some elements of Linear Algebra. Row reduction. Matrix addition, subtraction, multiplication, etc. Determinants. Commutators.

- *Physics Context - Kirchhoff's laws for circuit analysis. Numerical solutions to Laplace's equation. Commutation properties in QM. Calculation of cross products. Linear independence of solutions.*

Week 4 Continuation of Linear Algebra. Physical properties of matrices with physical relevance (transpose, conjugate, inverse, hermitian, singularity, etc.) Identification of orthogonal, Hermitian, Anti-Hermitian, and Unitary matrices. Rotations in 2d and 3d.

- *Physics Context - Measurability in QM. Euler Angles.*

Week 5 The Eigenvalue Problem. Calculation of eigenvalues and eigenvectors. Relationships between matrix properties and eigenvalue/eigenvector properties.

- *Physics Context - Pretty much the entire undergraduate Physics curriculum. Principle axes for solid-body rotation; principle axes for solid-body elasticity; optical polarization properties. Measureables and eigenfunctions in QM. Extensions of formalism to linear operators / Hilbert space. Coupled oscillations and normal modes of oscillation. Similarity transformations.*

Week 6 Review of Vector Calculus. Divergence/Gradient/Curl/Laplacian. Line integrals. Conservative fields. Helmholtz theorem. Divergence Theorem. Stokes' Theorem. Green's Theorem in a Plane.

- *Physics Context - Extremely broad. Geometrical interpretation of Maxwell's laws, Continuity equations, and Poisson's equation. Relationship between conservative fields and the notion of a scalar potential; finding associated scalar potentials. Important and extensive links to Classical Mechanics, E&M, Fluids, etc. Introduction of advective/material derivative and its Physical interpretation in different contexts. Introduction to Navier-Stokes' equation. Potential flow. Applications of Gauss' and Amepére's laws, etc. Many many more.*

Week 7-8 Index notation / Summation notation. Kronecker delta. Levy-Civita symbol. The delta-epsilon identity. Introduction to Cartesian Tensors.

- *Physics Context - Widely used in upper-level undergraduate and graduate texts. Standard notational treatment in solid state physics, GR, fluids, mechanics, E&M, etc. Writing vector identities. Matrix representation of curl. First Physical examples of tensors would likely include moment of Inertia tensor, atomic polarizability tensor, permittivity tensor, optical conductivity tensor, coherency tensor, field tensors in GR, stress-strain tensors, etc.*

Week 9-10 Introduction / Review of Differential Equations. Extremely simple introduction to solution methods (separable, Laplace transforms, series solutions, separation of variables). Identification of similarities between different Diff Eqs used in undergraduate Physics curriculum.

- *Physics Context - All over the place. This is not meant to replace a differential equations class in any sense of the word. Just enough to give students a flavor/Physics context. Emphasis is more on seeing how Heat Equation, Schrödinger Equation, and Diffusion equation are really all the same thing rather than how to solve them. Some introduction to Special Functions. Diff Eqs discussed to some degree: Beer-Lambert-Bourger Law/Radioactive Decay Law/Population Growth Models/Hydrostatic Balance equation; Hooke's Law/Newton's Second Law; Damped/Driven Harmonic Oscillator; Continuity Equation/Gauss Law, Faraday's Law, Ampére-Maxwell Law, Vorticity Equation, Laplace Equation, Poisson's Equation, Diffusion Equation, Schrödinger Equation for a Free Particle, Heat Equation, Wave Equation, Helmholtz Equation, Time-Dependent and Time-Independent Schrödinger Equation, Klein-Gordon Equation, Euler-Lagrange Equations, Hamilton's Equations, Logistic Equation.*

Week 11 Dirac Delta Function/Heaviside Function, Integral Transforms / Fourier Series (sin/cos, Bessel, Legendre, etc.), Assorted Special Functions.

- *Physics Context - Electricity and Magnetism; vector derivatives in Cylindrical and Spherical coordinates. Green's function methods. Convolutions (later linked to autocorrelation functions, etc.) Laplace transforms revisited. Applications to solutions using separation of variables method via completeness and orthogonality.*

Week 12 Fourier Series Cont'd and the Fourier Transform

- *Physics Context - Time-series analysis. Interpretation of experimental data. Links back to convolution via Wiener-Khinchine. Physical understanding of Dirichlet Conditions and extensions thereof. Parseval's theorem and its physical implications for Fourier Analysis. Use of even/odd symmetry. Gabor Limit and Heisenberg Uncertainty Principle. Wave packet localization. Numerical methods in data analysis. Power Spectral Density vs. $\mathscr{F}(\omega)$. Introduction to basic Fourier optics.*

Week 13-14 Probability and Statistics

- *Physics Context - Bayesian analysis, Partition Functions / Bose-Einstein/Fermi-Dirac/Maxwell-Boltzmann statistics. Basic introduction to probability density and distribution functions. Mean, Variance, Skewness, Kurtosis - Physical interpretation, how to calculate for both discrete and continuous r.v. Chebyshev's inequality and its use in calculating uncertainty. Introduction to important physically relevant probability distributions including discrete uniform, uniform, exponential, Gaussian, poisson, binomial. Generating Functions, Moments, Cumulants. Fitting data. Least-squares. Weighted least-squares. Shot noise. Basic error estimation. Interpretation, range of applicability, and appropriateness of the central limit theorem.*

- Other Possible Topics (to replace other ideas or to add if time):
  - Variational Calculus *Development of the Euler-Lagrange and Hamilton Equations*
  - Perturbation Theory *Can show up nearly anywhere. Most commonly seen at undergraduate level in Quantum Mechanics*
  - Wavelets *finite time-series analysis*
  - More extensive introduction to basic ODE/PDE solution methods (integrating factor, etc.) *useful throughout undergrad curriculum*
  - Elliptic Integrals *e.g. the Brachistochrone problem*
  - Others?