**Homework 7, PHYS 272 (Methods of Applied Physics)**
**Spring 2020**
**Due Friday, February 28th at Start of Class**

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

> As I have probably mentioned, I pride myself on not giving you a homework assignment unless we have already covered everything in class needed to complete the assignment; this assures that you have an entire week to complete the work if you start right away and don't have to wait until we cover something in lecture. At first glance, it might seem like this homework includes stuff that we have not talked about in class yet. The things we have not talked about in class yet, however, are thins you absolutely should have done in Calculus III. You might have to look some things up in other resources (and we'll be talking about these computations in class over the next few days), but you should not have to wait for our in-class review to start attacking these things.

> Another notational note. I will talk about this in class, but in case you missed it – we will be using the Griffiths notation for unit vectors in this class. Thus, the Cartesian unit vectors will be written $\hat{x}$, $\hat{y}$, and $\hat{z}$ (and NOT $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$). Unit vectors in cylindrical coordinates will be $\hat{s}$, $\hat{\varphi}$ and $\hat{z}$ (there is no $\hat{r}$ or $\hat{\theta}$ in cylindrical coordinates) and the unit vectors in spherical coordinates (also sometimes called spherical-polar coordinates) will be $\hat{r}$, $\hat{\theta}$, and $\hat{\varphi}$ (there is no $\hat{\rho}$ in spherical coordinates and $\theta$ corresponds to the angle of declination from the $z$-axis). Take care! This may not be the symbolic convention you are used to yet, but it is increasingly standard in Physics and we will be sticking to it in this class to help you prepare for E&M and/or Quantum Mechanics.

1. Use the vectors as defined below to compute the requested quantities. Leave all vector answers in Cartesian coordinates. Make sure to simplify everything and – remember – no calculators or technology!

$$\begin{aligned} \vec{A} &= 3\hat{x} + 4\hat{y} - 5\hat{z} \\ \vec{B} &= 2\hat{x} - 4\hat{z} \\ \vec{C} &\rightarrow s = 3, \varphi = \pi/6, z = 0 \\ \vec{D} &\rightarrow r = 4, \theta = \frac{\pi}{4}, \varphi = \frac{2\pi}{3} \end{aligned}$$

   a) $\vec{A} \cdot \vec{B}$, $\vec{B} \cdot \vec{C}$, $\vec{C} \cdot \vec{D}$, and $\vec{A} \cdot \vec{D}$.
   b) $\vec{A} \times \vec{B}$, $\vec{B} \times \vec{C}$, $\vec{C} \times \vec{D}$, and $\vec{D} \times \vec{A}$
   c) $\vec{A} \times (\vec{B} \times \vec{C})$ and $(\vec{A} \times \vec{B}) \times \vec{C}$

$$
\begin{aligned}
\vec{a} &= yz\hat{x} + xz\hat{y} + xy\hat{z} \\
\vec{b} &= x^2 e^{-z}\hat{x} + y^3\ln(x)\hat{y} + z\cosh(iy)\hat{z} \\
\vec{c} &= r^2\hat{r} \\
\vec{d} &= \frac{1}{s^2}\hat{s} \\
f &= 3x^2yz \\
g &= \cosh(xy) \\
h &= r^3
\end{aligned}
$$

2. Use the functions as defined above to compute the following. (Remember that the formulas for divergence, curl, gradient, and Laplacian for non-Cartesian coordinate systems may not be the obvious extension of their Cartesian counterparts. For example, you should not be using a determinant to calculate the curl of $\vec{c}$ or $\vec{d}$.)

   a) $\vec{\nabla}\cdot\vec{a}$, $\vec{\nabla}\cdot\vec{b}$, $\vec{\nabla}\cdot\vec{c}$, and $\vec{\nabla}\cdot\vec{d}$

   b) $\vec{\nabla}\times\vec{a}$, $\vec{\nabla}\times\vec{b}$, $\vec{\nabla}\times\vec{c}$, and $\vec{\nabla}\times\vec{d}$

   c) $\vec{\nabla}f$, $\vec{\nabla}g$, and $\vec{\nabla}h$

   d) $\nabla^2 f$, $\nabla^2 g$, and $\nabla^2 h$. In case you don't remember, $\nabla^2$ is called the Laplacian and writing $\nabla^2 f$ is another way of writing $\vec{\nabla}\cdot\left(\vec{\nabla}f\right)$

3. Consider the function $f(x, y) = 32 - x^2 - 4y^4$.

   a) What is $\vec{\nabla}f$?

   b) Using the above result, what is the gradient evaluated at the point $x = 3$, $y = -1$?

   c) If you were at the point $x = 3$, $y = -1$ and wanted to move in the direction of greatest instantaneous decrease of $f$, what direction (expressed as a 2-dimensional unit vector) would you move?