Assignment VIII, PHYS 230
Due Thursday, April 5 at start of class

Read Chapter 5 and 6 of your Text!

1. I asserted in class that any complex number \( z = x + iy \) (with \( x \) and \( y \) real) can be written as \( z = r \exp(i\theta) \) (with \( r \) and \( \theta \) real) [this is commonly called “polar-form”; we interpret \( r \) as the magnitude of the complex number and \( \theta \) an angle with respect to the real axis]. Demonstrate that these two forms are interchangeable by writing down:
   a) An expression for \( x \) in terms of \( r \) and \( \theta \)
   b) An expression for \( y \) in terms of \( r \) and \( \theta \)
   c) An expression for \( \theta \) in terms of \( x \) and \( y \)
   d) An expression for \( r \) in terms of \( x \) and \( y \)
   (Euler’s identity should come in handy).

2. Above, it was stated that when a complex number is in its polar form \( r \exp(i\theta) \), \( r \) corresponds to the magnitude of the complex number. In general, we define the magnitude of a complex number via:

\[
|z|^2 = z^* z = zz^*
\]

a) Show that \( |z|^2 = r^2 \). (Multiply \((x + iy)\) and \((x - iy)\), show that the product is consistent with the answer to question 1, part d).

b) Show that \( |\exp(i\theta)| = 1 \).

c) What would be the conjugate of \( r \exp(i\theta) \) (in terms of \( r \) and \( \theta \))?

(Important implication from the results of the above problem…..for any standing wave of the form \( \Psi(x,t) = \psi(x) \exp(-i\omega t) \), we can write \( |\Psi(x,t)| = |\psi(x)| |\exp(-i\omega t)| = |\psi(x)| \) and, therefore, the probability density \( |\Psi(x,t)|^2 dx = |\psi(x)|^2 dx \) is independent of time.)

3. (Problem 6-3 from your text) In a region of space, a particle has a wave function given by \( \psi(x) = A \exp(-x^2/2L^2) \) and energy \( \hbar^2/2mL^2 \), where \( L \) is some length. (a) Find the potential energy as a function of \( x \), and sketch (i.e. graph) \( V \) versus \( x \). (b) What is the classical potential that has this dependence?

4. (Problem 6-10 from your text) A particle is in the ground state of an infinite square well potential given by Equation 6-21:

\[
V(x) = \begin{cases} 
0 & 0 < x < L \\
\infty & x < 0 \text{ and } x > L 
\end{cases}
\]

Find the probability of finding the particle in the interval \( \Delta x = 0.002L \) at (a) \( x = L/2 \), (b) \( x = 2L/3 \), and (c) \( x = L \). (Since \( \Delta x \) is very small, you need not do any integration – the probability is effectively equal to \( \psi^*(x) \psi(x) \Delta x \).)
5. (Problem 6-11 from your text) Do problem 6-10 for a particle in the second excited state \((n = 3)\) of an infinite square well potential.

6. Use your answers from the above two problems to discuss the large \(n\) limit (i.e. what happens when the correspondence principle applies). What would you expect your answers to parts (a), (b), and (c) from the above two problems to be when \(n \to \infty\)?

7. (Problem 6-18 from your text) Suppose a macroscopic bead with a mass of 2.0 grams is constrained to move on a straight frictionless wire between two heavy stops clamped firmly to the wire 10 cm apart. If the bead is moving at a speed of 20 nanometers per year (i.e. to all appearances it is at rest), what is the value of its quantum number, \(n\)?

8. (Problem 6-56 from your text) For the wave functions:

\[
\psi(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n \pi x}{L} \right) \quad n = 1, 2, 3, \ldots
\]

corresponding to an infinite square well of width \(L\), show that:

\[
\langle x^2 \rangle = \frac{L^2}{3} - \frac{L^2}{2n^2 \pi^2}
\]

**Mathematica and/or MATLAB Problem**

9. The normalized eigenfunction solutions to the infinite square well potential are as follows:

\[
\psi_n(x) = \begin{cases} 
\sqrt{\frac{2}{L}} \sin \left( \frac{n \pi x}{L} \right) & 0 \leq x \leq L \\
0 & x < 0 \text{ or } x > L 
\end{cases}
\]

with the energies associated with each state equal to:

\[
E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2}
\]

a) Build a MATLAB or Mathematica function that takes in \(m\), \(L\), and \(n\) and outputs the energy associated with the level in Electron Volts!

b) Use MATLAB or Mathematica somehow to find what \(L\) must be for \(E_1\) for an electron to be 13.6 eV.

c) Use MATLAB or Mathematica to generate a function/interactive plot that allows the user (through the `Manipulate` command or via an argument to a function) to plot \(\psi_n(x)\) or \(\psi_n^*(x)\)\(\psi_n(x)\) (the user gets the choice). Of course, \(n\) must be an integer. You may set \(L = 1\) for convenience if you wish.