Assignment I, PHYS 230 (Introduction to Modern Physics)
Fall 2015
Due 8/27/15 at start of class

I suspect that the math and physics background of students in this class varies substantially. Therefore, this first homework is an attempt to find out what you've learned (and retained) from your previous courses.

This homework will be graded a bit unusually – I won't grade your answers for correctness; this time I'm only grading for effort / completeness. This homework will also be the only homework all semester that (1) I won't return to you, and (2) will count for less than a full homework grade.

Although I normally encourage you to work with classmates (so long as the work you turn in is your own), *on this homework, I do NOT want you to work with any of your classmates. Also, do not use calculators, computer software, or the internet to help you out! (I want to see what you remember, not what you can look up or be reminded of by a friend!)*

As always, what you get out of a class is really a function of how much you put into it. You could turn this homework in and use online resources and calculators to suggest you know more than you do – but then I'm going to assume that the class is better prepared than it is and probably move way too fast. Alternatively, you could just do a few problems and turn it in, but then I'll assume you've forgotten or never learned things that you really remember quite well and you'll be bored mindless – not exactly the best use of your hard earned cash. Therefore, it is in your best interest to put some effort into this, but to do it honestly (without using calculators, computer algebra systems like mathematica, or getting help from your fellow students).

Please put your answers on a separate sheet of paper. Try to make sure everything is legible and well organized.

1. List all Mathematics classes you've ever taken (High School or College), starting with Algebra I.
2. Tell me something you hope to get out of this class.
3. What is the value of $\cos \pi$?
4. What is the value of $\sin(60^\circ)$?
5. What is the value of $\tan\left(\frac{\pi}{3}\right)$?
6. Simplify $\sqrt{144a^2b^4c^7}$.
7. Evaluate: $\sqrt{-121}$
8. Evaluate $\sqrt[3]{-8}$
9. Evaluate $16^{-1/2}$
10. What is the volume of a spherical ball with diameter 3 cm?
11. What is the surface area of a spherical ball with diameter 3 cm?

12. A right triangle has a hypotenuse of length 25, and one of the other sides has length 24. What is the length of the remaining side?

13. What are the three angles inside the above described triangle? (Remember – no calculators! But an answer that looks like, say, $\sin^{-1}\left(\frac{3}{7}\right)$ is completely appropriate.)

14. Solve for $x : 5x^2 + 6x = -1$.

15. Find x and y so that both of these equations are satisfied:

$$8x + 4y = 16$$
$$6x - 3y = 24$$

16. Put the following ten numbers in order from smallest (closest to $-\infty$) to largest (closest to ∞): the number of seconds in a year, the number of molecules in a mole, $|3 \times 10^{-27}|$, π, 7×10^{-3}, -1, 0, 2×10^{17}, 5^3, $\frac{1}{100}$.

17. What is $\ln\left(e^{3\pi}\right)$

18. Find $\lim_{x \to 0}\left[\frac{\sin^2(3x)}{(2x)^2}\right]$

19. Calculate: $\frac{\mathrm{d}}{\mathrm{d}x}[3x^5]$

20. Calculate: $\frac{\partial}{\partial z}\left[\frac{4x^4}{y^3 z}\right]$

21. On some graph paper (or, if you don't have graph paper – on carefully constructed and measured axes), plot the equation $y = 3x + 5$.

22. On a different axes, plot the equation $y = |3x - 2| + 1$.

23. On a different axes, plot the equation $y = -3x^2 + 2$.

24. On a different axes, plot the equation $y = \frac{2x}{x^2 - x - 2}$.

25. Integrate: $\int x^2 \mathrm{d}x$

26. Integrate: $\int_0^{\pi/2} \cos x \mathrm{d}x$

27. Integrate $\int_0^1 x \exp(-x^2) \mathrm{d}x$

28. Vector $\vec{r}_1 = 3\hat{x} - 2\hat{y}$. Vector $\vec{r}_2 = -2\hat{y}$. What is $|\vec{r}_1 + \vec{r}_2|$?

29. Three men and three women (a total of 6 people) are prepared to enter a room. Three of these people enter the room. What is the probability that there are 2 men and 1 woman in the room?

30. Two urns contain colored balls. You are unable to look into the urns, but you know that one urn has a total of 11 balls – 7 red and 4 green. The other urn has 12 balls – 3 red and 9 green. You reach into one of the urns and pull out a red ball. What is the probability you pulled out of the first urn?

31. Evaluate the following: $(7\hat{x} + 6\hat{z}) \cdot (3\hat{x} + 2\hat{y} + \hat{z})$

32. Evaluate the following: $(7\hat{x} + 6\hat{z}) \times (3\hat{x} + 2\hat{y} + \hat{z})$

33. Evaluate the following: $\vec{\nabla}[5x^2y - \sin(xz) + 17]$

34. Evaluate the following: $\vec{\nabla} \cdot (4xz\hat{x} + 3yz\hat{y} + 4\hat{z})$

35. Evaluate the following: $\vec{\nabla} \times (4xz\hat{x} + 3yz\hat{y} + 4\hat{z})$

36. Simplify to an answer of the form $x + iy$: $\tanh \frac{3\pi i}{4}$.

37. What is the determinant of the following matrix?

$$\begin{pmatrix} 0 & 1 & 2 & -1 \\ -1 & 0 & -3 & 0 \\ -2 & 3 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

38. Find the eigenvalues and associated eigenvectors for the following matrix:

$$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

39. Evaluate $\oint \vec{V} \cdot \mathrm{d}\vec{r}$ around the boundary of the square with vertices $(1,0), (0,1), (-1,0), (0,-1)$ if $\vec{V} = x^2\hat{x} + 5x\hat{y}$.

40. Solve the following subject to the conditions $x(t=0) = 0$ and $x(t=1) = 3$:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 4x = 0$$

41. Solve the following (there will be an undetermined constant at the end):

$$\frac{\mathrm{d}^3z}{\mathrm{d}t^3} + 4\frac{\mathrm{d}z}{\mathrm{d}t} = 3t - 1$$

42. Solve the following subject to the boundary conditions $u(t,0) = 0$ and $u(t,L) = 0$. (The initial conditions are unspecified).

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

43. Are there any completely unfamiliar/incomprehensible problems on this assignment? Which one(s)?