Assignment I, PHYS 301 (Classical Mechanics)
Spring 2015
Due 1/16/15 at start of class

PLEASE READ THE COVER PAGE CAREFULLY – IT OUTLINES SOME BASIC PROTOCOLS FOR SOLVING HOMEWORK PROBLEMS IN THIS CLASS AND WON'T BE MENTIONED AGAIN!!!

In this class, you will be receiving weekly homework assignments. They are meant to be challenging, and likely will take a good chunk of your time. I apologize for this – but there's really no way around it. One learns Physics by actually solving problems. If you "get the ideas" but can't solve a problem associated with a particular concept, you have not yet mastered the concepts and tools you need to effectively "do" Physics. Everything always makes more sense when your professor is doing it on the board than when you are trying to do it yourself. For most, the best way to get better at solving problems is to get more practice. That's why we have the homework. (Trust me; I don't give this to you for my benefit.)

This course will be challenging, and this first homework is to ensure that you're really prepared for this course. Thus, the first assignment has some basic math at the level of MATH 323 (Differential Equations) and/or PHYS 272 (Methods of Applied Physics), as well as some questions that will hopefully be a good warm-up for the semester. The Physics problems do not require any knowledge beyond PHYS 111, but they might be a bit tricky.

The Math questions below are straightforward; if you struggle mightily with them – you may wish to ask yourself if you are really ready for this course.

Some other homework preliminaries that we need to get out of the way:

- *IN THIS COURSE, DO NOT USE MATHEMATICA, OTHER COMPUTATIONAL TOOLS, OR CALCULATORS UNLESS I SPECIFICALLY TELL YOU THAT YOU SHOULD!*
- Successful students historically have started working on the homework right after turning in their previous homework assignments; if you wait until the night before an assignment is due to start it, you will almost certainly struggle. Think of attacking this homework as a daily ritual – if you work on it a bit each day, it isn't nearly so daunting.
- I do not require you to solve problems any particular way – however, a quickly drawn sketch often will help in mechanics problems. I suggest you might want to try that to just picture what is going on before starting to push around symbols.
- You do not have to type your solutions – but it is appreciated if you do. I do expect that any solutions you give me are legible and easy to follow. When the problem asks you to find an expression, please *circle* or *box* the final answer so it is easier for me to find. Although it is not required, I do recommend you put each answer on a separate piece of paper. (My answer keys often will not put each answer on a spare sheet of paper, but that's to save a bunch of paper – I make 15-20 copies of each answer key, so saving half a page here and there really adds up in my case.)

- YOU MUST LEAVE ANSWERS IN TERMS OF THE VARIABLES GIVEN IN THE PROBLEM STATEMENT ONLY. If the problem refers to variables m, M, and a, only refer to m, M, and a in your solution. (You may, however, also include constants like 2, π, $\sqrt{2}$, g, G, R_E, etc.) I try to make the questions as clear as possible but, if you are in doubt, ask!

- Note – those of you who took other courses from me already probably realize this. Units are your friends. Checking the units of your final answer can help you see if you've made a mistake. (It might not tell you *where* you've made a mistake, but if your answer does not have the right units at the end, there must be a mix-up somewhere.) I use this trick all the time – and it can save you some serious anguish.

- More than anything else, please make your work clear and – especially if you aren't 100% sure you are correct – *EXPLAIN YOUR THOUGHT PROCESS!!!* I can't give you partial credit if I can't figure out what you did. PLEASE DO NOT JUST GIVE ME A LONG STRING OF EQUATIONS AS AN ANSWER! This isn't a Math class – we're talking about physical systems. An equation doesn't just come from nowhere. Give me context. Give me thoughts. Give me what ideas you are trying to use! You may earn a little bit more credit if you include this and – more importantly – if you give me TEXT in your answers, I can better help you figure out if you make a conceptual error. If all I have to grade are a bunch of equations right after each other in a row, it is difficult for me (or anyone else) to figure out what you were doing. I will always be happier if you include more words to describe your ideas/reasoning/thoughts. I know it takes a bit more work, but the more you show me about your lines of thought, the more effective I can be in helping you attack these problems. You don't necessarily have to write in complete sentences, but sometimes a word or two makes all the difference in helping me understand what you were trying to do. (Of course, you don't have to take this to extremes – if you divide both sides of an equation by 2, I can probably figure that out without any text..... when in doubt, include more words. It can't hurt.)

- You may work with your classmates on the homework, but it is expected that your final solutions are your own. Collaborate/discuss/work out elements of the problem with each other if you wish – but when it comes time to write your actual solution, don't copy from each other – get the basic ideas of what you have to do and then go off and write your own answers.

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Part I: Straight-Up Math

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1. Integrate $\int_0^4 x \exp(-x^2) \mathrm{d}x$
2. Solve the following differential equation: $r \frac{\mathrm{d}r}{\mathrm{d}t} = 3$ with $r(t = 0) = r_\circ$.
3. Solve the following differential equation: $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + k^2 x = 0$ with $x(t = 0) = A$ and $\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} = B$. You may assume $k > 0$.
4. What is the determinant of the following matrix?

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

5. Find the eigenvalues and normalized associated eigenvectors for the following matrix:

$$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

6. What is the surface area of a spherical ball with diameter 4 cm?

7. For $x \ll 1$, approximate $f(x) = \left(\frac{\sqrt{1+2x}}{\sqrt{1-x}} - 1\right)$?

   [Note $f(x) \approx 0$ is *NOT* the right answer here. Sure – the answer is small – but it isn't actually zero unless $x = 0$. Whenever you are asked to approximate something, you should keep the largest non-vanishing terms – never report the value as 0 unless the value is *exactly* 0. Here, I'm happy if you just keep the largest non-vanishing term. (As a hint, I'll tell you – the largest nonvanishing term here is linear in x – your answer should be αx with α some constant you are tasked to find.)]

**** PHYS 111 enhanced ****

8. You only need to consider units to do the following problem. Consider a vibrating water drop, where the vibrational frequency ν may depend on the drop radius R, the drop mass density ρ, and the drop surface tension σ. (The units of a surface tension are energy/area). Assume $\nu \propto R^\alpha \rho^\beta \sigma^\gamma$ with α, β, and γ real constants. Find α, β, and γ. (This is an extremely powerful technique).

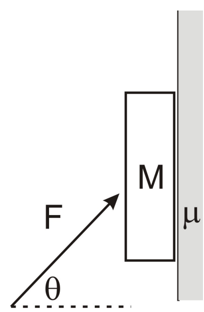
9. This one is a deceptively tricky thought experiment. Suppose you tranquilize a polar bear on a frictionless glacier as part of a research effort. How can you estimate the mass of the bear using a measuring tape, a rope (with negligible mass), and a knowledge of your own mass? In a few sentences, describe the procedure you would use, the measurements you would make, and show how the measured quantities enable you to determine the mass of the bear. Hints/reminders: remember that the surface of the glacier is presumably flat and – given the parameters of the problem – frictionless. Presumably you know (or can measure) the length of the rope. Also note that you don't have a stopwatch or any other reliable timing mechanism.

10. A book of mass M is positioned against a vertical wall. The coefficient of friction between the wall and the book is μ. (We'll set the coefficient of static friction and the coefficient of kinetic friction to be the same, for simplicity). You wish to keep the book from falling by pushing on it with a force F applied at an angle θ with respect to the horizontal as shown below. θ is automatically constrained between $-\pi/2$ and $\pi/2$ from the geometry of the problem.

    a) For a given θ, what is the minimum F required?

    b) For what θ is this minimum F the smallest?

    c) Is there a value of θ, below which there does not exist an F that keeps the book up? If so, find it.



11. Three masses accelerate to the right as shown below. The rightmost mass has mass M, and the other two masses are m_1 and m_2 as shown. The three masses are connected by ropes of negligible mass. The masses slide on a frictionless surface. The rightmost mass accelerates with constant acceleration a. The entire force generating this motion is not shown, but acts directly on mass M only. (The other masses accelerate because they are pulled by M). Determine:

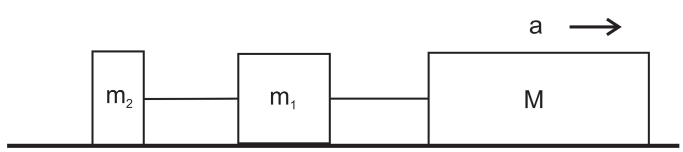
    a) The magnitude of the force applied to M to make this system move.

    b) The tension in the rope between M and m_1.

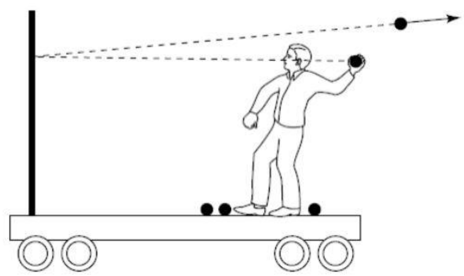
    c) The tension in the rope between m_1 and m_2.

    d) The net force on m_1.

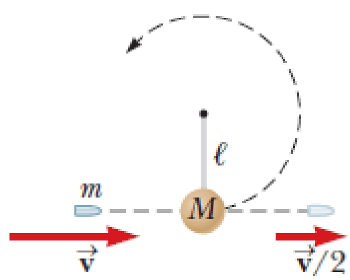
    (Remember, give all answers in terms of m_1, m_2, M, and a only!!!!)



12. Suppose you are on a cart, initially at rest on a track with negligible friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back at you as shown in the figure below, is the cart put in motion? If so, which way? (To the left or to the right?) Explain.



13. A bullet of mass m and initial speed v passes completely through an initially stationary pendulum bob of mass M and length ℓ. The bullet emerges with a speed αv. (α is an unspecified constant somewhere between 0 and 1). The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle? (Note, the picture below assumes $\alpha = \frac{1}{2}$, but we want to keep the answer general here).



14. A thin uniform rod of mass M and length L is positioned vertically above an anchored frictionless pivot point, as shown below, and then allowed to fall to the ground. With what speed does the free end of the rod strike the ground? [Hint – you might want to either calculate or look up the moment of inertia of a rod].