

**Homework 1, PHYS 415 (Fluid Mechanics)**  
**Spring 2019**  
**Due Thursday 10th January 2019 at Beginning of Class**

The background of students in this class is highly variable. As I write this, there are 12 students enrolled – nearly all of you are (at least) double majors, and there are folks taking this class for a variety of reasons and who ultimately need to learn a variety of different things.

Despite the fact that Differential Equations and/or Methods of Applied Physics is a prerequisite for this class, I fully expect the level of mathematical preparedness to vary a bit from student to student. This first homework is (1) to verify you know the things you need to know to succeed in this class, (2) find out if there are any general holes in the classes' mathematical knowledge, and (3) to get our mental juices flowing after the break.

There is nothing on this homework that you shouldn't be able to do without external resources. *YOU SHOULD NOT NEED A CALCULATOR OR MATHEMATICA TO DO ANY OF THESE PROBLEMS! AS SUCH, YOU ARE ON YOUR HONOR TO NOT USE THEM – OR OTHER AIDS – TO DO THIS WORK.* Also, though I normally encourage you to work with others on homework assignments, please do this one on your own.

Please put your answers on a separate sheet of paper. Make sure everything is legible and well organized. (Illegible work will not be counted for full credit).

1. List all Mathematics classes you've ever taken (High School and College), starting with Calculus I.
2. List all Physics courses you've taken since High School. If you took it at a different institution, please give me the course name instead of the course number.
3. Tell me something you hope to get out of this class.
4. Solve for  $x$ :  $6x + 7 = 55$
5. Solve for  $x$ :  $6(x + 7) = 54$
6. What is the value of  $\cos \pi$ ?
7. What is the value of  $\sin(60^\circ)$ ?
8. What is the value of  $\tan\left(\frac{\pi}{3}\right)$ ?
9. Simplify  $\sqrt{144a^2b^4c^7}$ .
10. Evaluate  $\sqrt[3]{-8}$
11. Evaluate  $16^{-1/2}$
12. A right triangle has a hypotenuse of length 25, and one of the other sides has length 24. What is the length of the remaining side?

13. Solve for  $x$ :  $5x^2 + 6x = -1$ .

14. Find  $x$  and  $y$  so that both of these equations are satisfied:

$$8x + 4y = 16$$

$$6x - 3y = 24$$

15. Evaluate:  $\sqrt{-121}$

16. What is  $\ln(e^{3\pi})$

17. On some graph paper (or, if you don't have graph paper – on carefully constructed and measured axes), plot the equation  $y = 3x + 5$ .

18. On a different axes, plot the equation  $y = |3x - 2| + 1$ .

19. On a different axes, plot the equation  $y = -3x^2 + 2$ .

20. On a different axes, plot the equation  $y = \frac{2x}{x^2 - x - 2}$ .

21. What is the volume of a spherical ball with diameter 3 cm?

22. What is the surface area of a spherical ball with diameter 3 cm?

23. Find  $\lim_{x \rightarrow 0} \left[ \frac{\sin^2(3x)}{(2x)^2} \right]$

24. Calculate:  $\frac{d}{dx}[3x^5]$

25. Integrate:  $\int x^2 dx$

26. Integrate:  $\int_0^{\pi/2} (\cos x) dx$

27. Calculate:  $\frac{\partial}{\partial z} \left[ \frac{4x^4}{y^3 z} \right]$

28. Put the following ten numbers in order from smallest (closest to  $-\infty$ ) to largest (closest to  $\infty$ ): the number of seconds in a year, the number of molecules in a mole,  $|-3 \times 10^{-27}|$ ,  $\pi$ ,  $7 \times 10^{-3}$ ,  $-1$ ,  $0$ ,  $2 \times 10^{17}$ ,  $5^3$ ,  $\frac{1}{100}$ .

29. Integrate  $\int_0^1 x \exp(-x^2) dx$

30. A vector  $\vec{r}$  has  $|\vec{r}| = 10$  and points  $30^\circ$  below the  $x$ -axis. What is the vector's  $y$ -component?

31. A vector is written  $-3\hat{i} - 3\hat{j}$ . Write the vector in polar coordinates.

32. Vector  $\vec{r}_1 = 3\hat{i} - 2\hat{j}$ . Vector  $\vec{r}_2 = -2\hat{j}$ . What is  $|\vec{r}_1 + \vec{r}_2|$ ?

33. Evaluate the following:  $(7\hat{i} + 6\hat{k}) - (3\hat{i} + 2\hat{j} + \hat{k})$

34. Evaluate the following:  $(7\hat{i} + 6\hat{k}) \cdot (3\hat{i} + 2\hat{j} + \hat{k})$

35. Evaluate the following:  $(7\hat{i} + 6\hat{k}) \times (3\hat{i} + 2\hat{j} + \hat{k})$
36. Three blond-haired people and three red-haired people (a total of 6 people) are prepared to enter a room. Three of these people enter the room. What is the probability that there are 2 blond-haired people and 1 red-haired person in the room?
37. Two boxes contain colored balls. You are unable to look into the boxes, but you know that one box has a total of 11 balls – 7 red and 4 green. The other box has 12 balls – 3 red and 9 green. You reach into one of the boxes and pull out a red ball. What is the probability you pulled out of the first box?
38. Evaluate the following:  $\vec{\nabla}[5x^2y - \sin(xz) + 17]$
39. Evaluate the following:  $\vec{\nabla} \cdot (4xz\hat{i} + 3yz\hat{j} + 4\hat{k})$
40. Evaluate the following:  $\vec{\nabla} \times (4xz\hat{i} + 3yz\hat{j} + 4\hat{k})$
41. Find the eigenvalues and associated eigenvectors for the following matrix:

$$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

42. Evaluate  $\oint \vec{V} \cdot d\vec{r}$  around the boundary of the square with vertices  $(1, 0), (0, 1), (-1, 0), (0, -1)$  if  $\vec{V} = x^2\hat{i} + 5x\hat{j}$ .
43. Solve the following subject to the conditions  $x(t = 0) = 0$  and  $x(t = 1) = 3$ :

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 0$$

44. Solve the following for  $z(t)$  (there will be an undetermined constant at the end):

$$\frac{d^3z}{dt^3} + 4\frac{dz}{dt} = 3t - 1$$

45. Solve the following subject to the boundary conditions  $u(t, 0) = 0$  and  $u(t, L) = 0$ . (The initial conditions are unspecified).

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$