Assignment II, PHYS 308
Fall 2014
Due 9/5/14 at start of class

NOTE: Just like last homework, please leave your answers in terms of actual numbers (with appropriate units) when appropriate.

Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't read it (or I can't follow your reasoning). Extensive exposition on your thought process or strategy is always appreciated.

1. You emit more energy (per unit area) in the infrared than Charon (a satellite of the dwarf planet Pluto). However, Charon is a bit bigger than you are. Assume the albedo of Charon is 0.37 independent of wavelength and that you are a perfect blackbody. Assume that Charon is spherical with radius 6.03×10^5 m. Calculate:

   a) The ratio:

   $$\frac{\text{Surface Area of Charon}}{\text{Surface Area of You}}$$

   b) The equilibrium temperature of Charon (assuming that it is a grey-body with albedo *for all wavelengths* equal to 0.37). Assume its only energy input is from the sun (at 6000K). Use 7×10^8 m as the radius of the Sun and the Sun-Charon distance to be the mean orbital Sun-Pluto radius of about 5.87×10^{12} m.

   c) The above calculation is a bit off (for various reasons). For the rest of this problem use the temperature of Charon to be 53K. Find the ratio:

   $$\frac{\text{Total Radiated Power Emitted by Charon}}{\text{Total Radiated Power Emitted by You (isolated in space)}}$$

   d) Find the ratio:

   $$\frac{\text{Total Spectral Emissions of Charon between 1 and 1.0001 } \mu\text{m}}{\text{Total Spectral Emissions of you Between 1 and 1.0001 } \mu\text{m (isolated in space)}}$$

2. Assume that the Venusian atmosphere is pure CO_2 (more or less true). Also assume the temperature of the surface gas on Venus matches the planet's mean surface temperature of about 740K. The radius of Venus is about 6.05×10^6 m and its mass is about 4.87×10^{24} kg. Find the scale-height (the height above the surface where the pressure is e^{-1} of its surface pressure) in the Venusian atmosphere. [Note – you will not need to know anything about how much actual gas is there to find this! Cool.]

3. A hypothetical star has an emissivity ϵ that depends on the star's radius via the function $\epsilon(r) = \frac{r}{R}$ with $0 \leq r \leq R$. (We will assume that for this star, ϵ is independent of wavelength). Assume that this star is somehow constrained to always emit the exact same total power, no matter what its current radius may be. From this information, you should be able to derive an expression for $T(r)$ for this star. Your task: Find $\frac{T(R/2)}{T(R/4)}$.

4. There is a general tendency to focus on the peaks of the black-body curve and ignore the tails. (If you talked about Wien's displacement law in Modern Physics, for example, you know easily how to calculate λ_{peak} as a function of temperature for a black-body). The peak is important – as is the general shape – but people often forget or overlook the T^4 dependence in the total radiated flux. It may be surprising to know that, even though Earth's blackbody peak is around 10μm, the sun still emits more 10 μm energy than the Earth does! (And that's even based on a "per-area" calculation, not just total. The sun emits more 10 μm energy per meter squared of the sun's surface than the Earth emits per meter squared at the Earth's surface). Let's reinforce this a bit with calculation.

   a) Calculate the ratio:

   $$\frac{I\left(10\mu\text{m}, T_{6000\text{K}}\right)}{I\left(10\mu\text{m}, T_{300\text{K}}\right)}$$

   (which is approximately the per-unit-area emission of the sun at 10 micron wavelengths to the per-unit-area emission of the Earth at 10 micron wavelengths. Notice that the answer is bigger than 1).

   b) Calculate the ratio of the total 10 micron emission of the sun (including an accounting of the sun's size) to the total 10 micron emission of the Earth.

   c) In our energy balance equations, we assumed that the incoming long-wave radiation from the sun (where 10 micron radiation would be classified) was negligible compared to the outgoing long-wave radiation from the Earth. Given your answers to (a) and (b), you might think this is a bad idea. Why isn't it a bad idea to make this assumption?

5. In class, Dr. Larsen tried "deriving" the dry adiabatic lapse rate for an ideal diatomic gas, but he left the last few steps for you to finish off. The last expression he wrote was:

   $$V_f = V_i \left(1 - \frac{z}{H}\right)^{-1/\gamma}$$

   Using $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$, the above expression, and your knowledge about γ for this sort of a gas, show that an initial 300K gas raised to 1 km in an atmosphere where $H \sim 8500$ meters loses about 10K. (Hint: recall, for $x < 1$ that $(1+x)^n \sim 1 + nx$)