Assignment II, PHYS 230 (Introduction to Modern Physics)
Fall 2015
Due 9/3/15 at start of class

As always, please put your answers on separate paper.

1. Before we get to the problems, I have another assessment for you. This is still a "homework problem", but if you complete it, you'll get full credit on this problem. Answer the problem honestly – I'm trying to find out what kind of scientific computer skills you have as a class.

   a) If you we given a function (say $f(x) = \left(\frac{\pi^4}{90}\sin^4(2\pi x)\right)^{-1/3}$ over the interval $x \in (0,1)$), would you currently be able to make a plot of this function with the following tools (just answer "yes I could" or "no, not yet" for each):

      i) Excel?
      ii) Open Office?
      iii) MATLAB?
      iv) Mathematica?
      v) Maple?
      vi) IDL?
      vii) Other? (If yes, please list all the computational tools you could use to complete this task right now).

   b) Let us say that, hypothetically, you are given an integral that you are not able to solve (but that you do know has a closed-form solution).

      i) Which computational tools from part (a) above do you think someone (not necessarily you) could theoretically use to solve the integral?
      ii) Do you currently know how to use any of the tools listed in part (a) in order to solve an integral? If yes, which ones?

   c) What is your familiarity level with LaTeX on a scale of 1-10. (1 = "what is LaTeX?"; 3 = "I know what LaTeX is, but I've never really done anything with it; 5 = "I've compiled a few simple documents, but I don't use it regularly"; 7 = "I can and have written lab reports with LaTeX"; 9 = "I know enough about LaTeX to know why mixing .jpg and .eps files is a bad idea, and I know the pain of trying to put a figure where I want it to go"; 10 = "I write my shopping lists with LaTeX".)

d) Have you ever written a computer program (in any computer language)? IF so, which language(s) do you have experience with? For each, indicate your proficiency level (beginner, basic competence, intermediate, or expert).

e) Have you ever written a computer program to simulate a scientific process? If so, which language(s) did you use?

f) Do you know basic UNIX commands?

g) Could you use a computer to find values of x so that $x^2 + 5x = \sin x$ is true? (Approximate values would be ok). If so, what computational tool(s) would you use?

h) Have you ever used a computational tool to solve a differential equation? If so, which tool(s) did you use?

i) Could you currently write a computer program (in any language) to convert Cartesian Coordinates into Cylindrical and Spherical Coordinates? If so – which one(s)?

j) Have you ever used LabVIEW, MATLAB, IDL, or any other scientific programming environment to acquire and analyze experimental data? If so, which tools have you used?

2. Your instructor skipped some steps in getting to the final Lorentz transformation (in the spherical light wave approach). Given his intermediate step:

$$x^2 - c^2t^2 = [\lambda(x - ut)]^2 - c^2 \left[\frac{(1-\lambda^2)x}{\lambda u} + \lambda t\right]^2$$

equate coefficients of the t^2 term to show

$$\lambda = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

[hint, you're going to have to FOIL and then do some algebra].

3. BACKGROUND: A tool very frequently used in your upper-level Physics classes is the binomial approximation. You almost certainly have seen this before, but it may not have been emphasized in your previous exposure. A brief review might help.

   If you have a quantity like $(a+b)^n$, and you know that a is larger than b, you can rewrite this as: $a^n\left(1+\frac{b}{a}\right)^n$. Why is this useful? Because you know that $b/a < 1$. When you have expressions that look like $(1+x)^n$ with $x < 1$, you can write:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \dots$$

   Actually, you can always write this. The advantage when $x < 1$ is that these terms usually rapidly decrease in magnitude, and a good approximation can usually be made in just keeping the first 2 terms $(1+nx)$. (If you want to be safe, keeping the third term isn't a bad idea). Thus, in the example above, for an expression like $(a+b)^n$ with $a > b$, we can write:

$$(a+b)^n = a^n\left(1+\frac{b}{a}\right)^n \sim a^n\left(1+\frac{bn}{a}+\frac{n(n-1)}{2}\left(\frac{b}{a}\right)^2\right) = a^n + nba^{n-1} + \frac{n(n-1)}{2}b^2a^{n-2}$$

   Why is this relevant here? In relativity, we get a lot of expressions with $(c^2-u^2)^{1/2}$ involved somewhere. Since u is generally much less than c, approximating these expressions for $u \ll c$ will frequently involve the binomial expansion.

   Your task: Approximate each of these expressions for $u \ll c$. In your final answer, keep the first two non-zero terms for each answer.

   a) γ. If you forgot, $\left(\gamma \equiv \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\right)$.

   b) γ^{-1}

   c) $1-\frac{1}{\gamma}$

   d) $\frac{1}{1+\frac{u}{c}}$

   e) $\sqrt{\frac{2}{c^2-u^2}+\frac{2}{c^2+u^2}}$

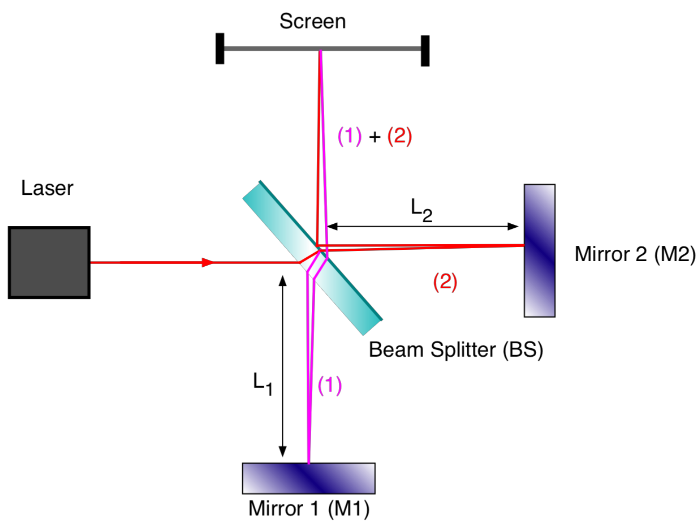

Figure 1: A simplified schematic of a Michelson interferometer.

4. In class, we discussed the Michelson-Morley experiment. Let's revisit that here. Below, see a basic schematic of a Michelson interferometer.

   Let $L_1 = L_2$ (which we'll just call L) and neglect the thickness of the beam splitter, so that both light beams travel the exact same distance from the laser to the screen. Let's assume, for this problem, that Galilean relativity really *does* work for light, and that there is an "ether" that is moving in the direction up the page (from mirror 1 to the screen) with velocity u. [Note – this should go without saying – this problem is *not an accurate depiction of reality!!* Rather, we are trying to explore reality as it would have been if the Michelson-Morley experiment actually detected an ether].

   a) For simplicity, let us assume the light moving to the right (towards mirror 2) and back to the left (after hitting mirror 2 and heading back towards the beam splitter) is not influenced by the ether. [This is admittedly weird; we're letting the ether influence the light moving vertically but not horizontally. We'll fix that in part(e) below.] For this case, the amount of time it takes for the beam to go from the beam splitter to mirror 2 and back to the beam splitter again is $\frac{2L}{c}$. How long does it take the light beam to go from the beam splitter to mirror 1 and back? (Remember, we're assuming Galilean relativity, so the speed at which the light gets from the beam splitter to mirror 2 is different than the speed at which it goes from mirror 2 back to the beam splitter).

b) Which beam returns to the beam splitter first? The beam that hits mirror 1 or the beam that hits mirror 2? Justify your answer.

c) Let's assume that $u = \frac{c}{4}$. How much of a delay will the slower beam be behind the faster beam when returning to the beam splitter? (Leave your answer in terms of L and c).

d) If L was 10 meters (close to the truth for the Michelson-Morley experiment) and $u = \frac{c}{4}$ (use $c \sim 3.00 \times 10^8$ m/s), what does your answer in part (c) come out to numerically? (Leave your answer in seconds).

e) If there *were* an ether, then the light beam moving towards mirror 2 would also be influenced a bit. We want the light to move directly to the right and left in the diagram, but the light would actually be dragged towards the screen by the ether; thus, the light path would have to be directed slightly down the page (towards mirror 1) so that the sum of the two velocity vectors yields a vector that points directly from the beam splitter to mirror 2. (See, e.g., example 2.3 in your text for a similar line of reasoning). Given this revised understanding of the path of the light beam moving to and from mirror 2, how long will it take the light beam to go from the beam splitter to mirror 2 and back? (Leave your answer in terms of L, u, and c.

f) Calculate the ratio $t_{\text{mirror 1}}/t_{\text{mirror 2}}$ when taking the effect in part (e) into account. (Your answer should look a bit familiar).