Assignment II, PHYS 230 (Modern Physics)
Fall 2019 Due August 29th, 2019 at Start of Class

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

1. We'll start out with something rather straightforward. See figure 1 (shamelessly cribbed from your text). If v is steady at 5 m/s, each boat always travels with a speed (relative to the water) of 13 m/s and L is 3.5 km. [Hint: look at your textbook if you need help; the problem where I grabbed the figure from gives lots of help.]

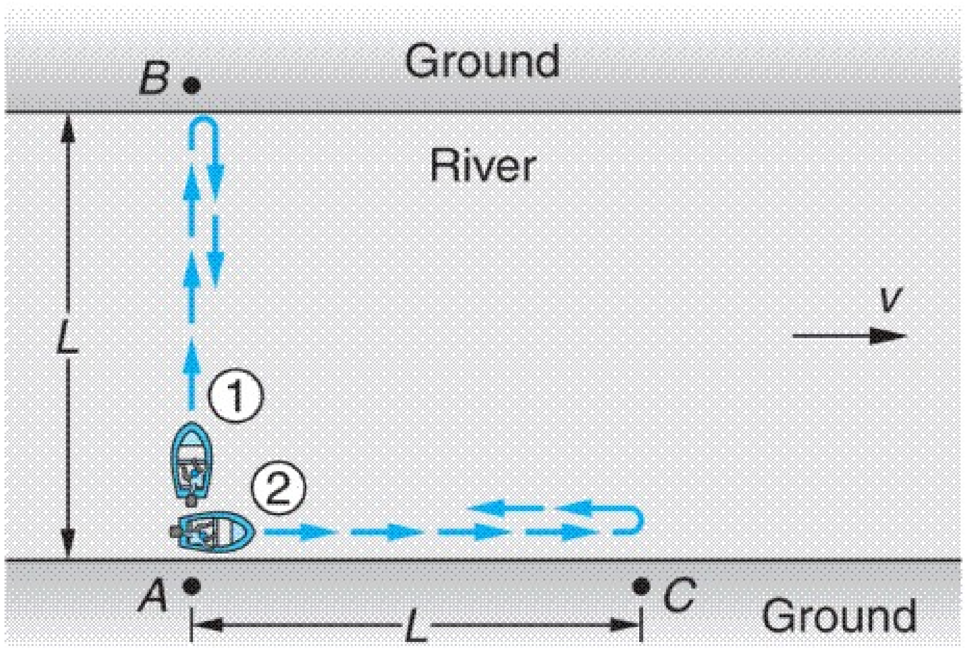

Figure 1: The figure associated with problem 1.

a) How long does it take boat 1 to return to point A?
b) How long does it take boat 2 to return to point A?
c) Which boat is faster, and by how much?

2. Your instructor skipped some steps in getting to the final Lorentz transformation (in the spherical light wave approach). Given his intermediate step:

$$x^2 - c^2 t^2 = [\lambda(x - vt)]^2 - c^2 \left[\frac{(1-\lambda^2)x}{\lambda v} + \lambda t \right]^2$$

equate coefficients of the t^2 term to show

$$\lambda = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

[hint, you're going to have to FOIL and then do some algebra].

3. In class, we either have derived (or are in the process of deriving) the Lorentz Transformation. The final result of this transformation reads:

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2) \end{aligned}$$

With $\gamma \equiv \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$. For this problem, let v (the relative velocity between the primed and the unprimed frame) be equal to $3c/5$.

a) What is the value of γ for this relative speed between the frames?

b) If an event (E1) occurs at the coordinates $x = 3.1$ m, $y = 2.3$ m, $z = -3.4$ m, and $t = 10$ ns, what are the coordinates of the event E1 in the primed frame?

c) If a different event (E2) occurs at coordinates $x = -2.1$ m, $y = -7.1$ m $z = 0$ m, and $t = 7.5$ ns, what are the coordinates of event E2 in the primed frame? [You should notice that in the primed frame, event 2 occurs after event 1, even though in the unprimed frame event 1 occurs after event 2!]

d) If an event (E3) occurs at the origin of the primed frame at $t' = 10$ s, at what coordinates did the event occur in the unprimed frame?

4. Your task: Approximate each of these expressions for $u \ll c$. In your final answer, keep the first two non-zero terms for each answer.

   a) γ. If you forgot, $\left(\gamma \equiv \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\right)$

   b) γ^{-1}

   c) $1 - \frac{1}{\gamma}$

   d) $\frac{1}{1+\frac{u}{c}}$

   e) $\sqrt{\frac{2}{c^2-u^2} + \frac{2}{c^2+u^2}}$

5. Read the summary of the Michelson-Morley Experiment at http://larsenml.people.cofc.edu/MichelsonMorley.pdf and watch the video at http://tinyurl.com/micmorvideo. Based on these sources, answer the following questions.

   a) Based on an equation in the first link, if you used $\epsilon_\circ = 8.854 \times 10^{-12}\ \mathrm{C^2/(N\ m^2)}$ and $\mu_\circ = 4\pi \times 10^{-7}\ \mathrm{N\ s^2/C^2}$, what value would you get for c? (You may use a calculator; make sure to keep at least 4 sig figs).

   b) If we treated "empty" space as a solid with a density associated with 1 hydrogen atom per cubic centimeter and we (erroneously) assumed that the aether exists, what would you expect the bulk modulus of space to be to get a speed of light moving at 3.00×10^8 m/s? (Make sure to include correct units; you may use a calculator. You might need to know that 6.02×10^{23} atoms of Hydrogen has a mass of about 1.0079 grams.

6. Let's continue to investigate the Michelson-Morley experiment. On the following page is a basic schematic of a Michelson interferometer that you may want to refer to as you attack this question.

   Let $L_1 = L_2$ (which we'll just call L) and neglect the thickness of the beam splitter, so that both light beams travel the exact same distance from the laser to the screen. Let's assume, for this problem, that Galilean relativity really *does* work for light, and that there is an "ether" that is moving in the direction up the page (from mirror 1 to the screen) with velocity u. [Note – this should go without saying – this entire problem is *not an accurate depiction of reality!!* Rather, we are trying to explore reality as it would have been if the Michelson-Morley experiment actually detected an ether].

   a) For simplicity, let us assume the light moving to the right (towards mirror 2) and back to the left (after hitting mirror 2 and heading back towards the beam splitter) is not influenced by the ether. [This is admittedly weird; we're letting the ether influence the light moving vertically but not horizontally. We'll fix that in part(e) below.] For this case, the amount of time it takes for the beam to go from the beam splitter to mirror 2 and back to the beam splitter again is $\frac{2L}{c}$. How long does it take the light beam to go from the beam splitter to mirror 1 and back?

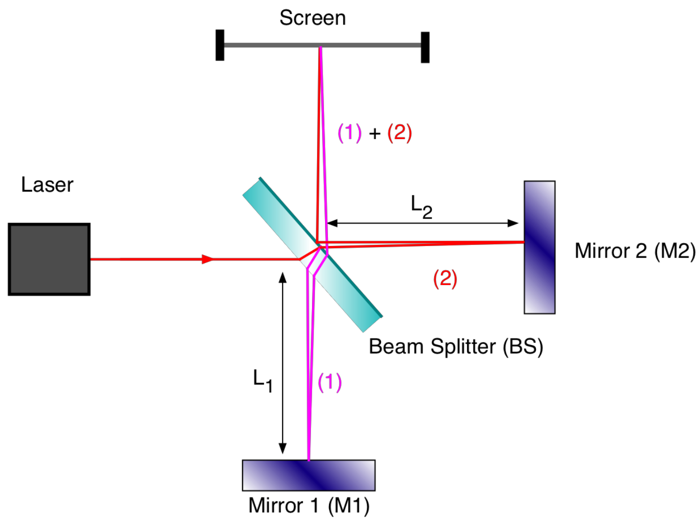

Figure 2: A simplified schematic of a Michelson interferometer.

(Remember, we're assuming Galilean relativity, so the speed at which the light gets from the beam splitter to mirror 2 is different than the speed at which it goes from mirror 2 back to the beam splitter).

b) Which beam returns to the beam splitter first? The beam that hits mirror 1 or the beam that hits mirror 2? Justify your answer.

c) Let's assume that $u = \frac{c}{4}$. How much of a delay will the slower beam be behind the faster beam when returning to the beam splitter? (Leave your answer in terms of L and c).

d) If L was 10 meters (close to the truth for the Michelson-Morley experiment) and $u = \frac{c}{4}$ (use $c \sim 3.00 \times 10^8$ m/s), what does your answer in part (c) come out to numerically? (Leave your answer in seconds; you may use a calculator for this part).

e) If there *were* an ether, then the light beam moving towards mirror 2 would also be influenced a bit. We want the light to move directly to the right and left in the diagram, but the light would actually be dragged towards the screen by the ether; thus, the light path would have to be directed slightly down the page (towards mirror 1) so that the sum of the two velocity vectors yields a vector that points directly from the beam splitter to mirror 2. Given this revised understanding of the path of the light beam moving to and from mirror 2, how long will it take the light beam to go from the beam splitter to mirror 2 and back? (Leave your answer in terms of L, u, and c.

f) Calculate the ratio $t_{\text{mirror 1}}/t_{\text{mirror 2}}$ when taking the effect in part (e) into account. (Your answer should look a bit familiar).