Assignment II, PHYS 301 (Classical Mechanics)
Spring 2014
Due 1/17/14 at start of class

PLEASE READ THE COVER PAGE CAREFULLY – IT OUTLINES SOME BASIC PROTOCOLS FOR SOLVING HOMEWORK PROBLEMS IN THIS CLASS AND WON'T BE MENTIONED AGAIN!!!

In this class, you will be receiving weekly homework assignments. They are meant to be challenging, and likely will take a good chunk of your time. I apologize for this – but there's really no way around it. One learns Physics by actually solving problems. If you "get the ideas" but can't solve a problem associated with a particular concept, you have not yet mastered the concepts and tools you need to effectively "do" Physics. Everything always makes more sense when your professor is doing it on the board than when you are trying to do it yourself. For most, the best way to get better at solving problems is to get more practice. That's why we have the homework. (Trust me; I don't give this to you for my benefit.)

Successful students historically have started working on the homework right after turning in their previous homework assignments; if you wait until the night before an assignment is due to start it, you will likely struggle. Think of attacking this homework as a daily ritual – if you work on it a bit each day, it isn't nearly so daunting. It is also worth mentioning that you can always get a bit of help from me. However, assignments are due on Friday and my last formal office hour of the week is on Tuesday. If you can find me, I'm happy to help later in the week – but I have a packed schedule and may not be around as much as you would hope. To be assured of getting help in time, attack the problems early so that you can get help in the early part of the week during office hours!

I do not require you to solve problems any particular way – however, a quickly drawn sketch often will help in mechanics problems. I suggest you might want to try that to just picture what is going on before starting to push around symbols.

You do not have to type your solutions – but it is appreciated if you do. I do expect that any solutions you give me are legible and easy to follow. When the problem asks you to find an expression, please *circle* or *box* the final answer so it is easier for me to find. Although it is not required, I do recommend you put each answer on a separate piece of paper. (My answer keys often will not put each answer on a spare sheet of paper, but that's to save a bunch of paper – I make 15-20 copies of each answer key, so saving half a page here and there really adds up in my case.)

PLEASE LEAVE ANSWERS IN TERMS OF THE VARIABLES GIVEN IN THE PROBLEM STATEMENT ONLY. If the problem refers to variables m, M, and a, only refer to m, M, and a in your solution. (You may, however, also include constants like 2, π, $\sqrt{2}$, g, G, R_E, etc.) I try to make the questions as clear as possible but, if you are in doubt, ask!

Note – those of you who took other courses from me already probably realize this. Units are your friends. Checking the units of your final answer can help you see if you've made a mistake. (It might not tell you *where* you've made a mistake, but if your answer does not have the right units at the end, there must be a mix-up somewhere.) I use this trick all the time – and it can save you some serious anguish.

More than anything else, please make your work clear and – especially if you aren't 100% sure you are correct – *EXPLAIN YOUR THOUGHT PROCESS!!!* I can't give you partial credit if I can't figure out what you did. PLEASE DO NOT JUST GIVE ME A LONG STRING OF EQUATIONS AS AN ANSWER! This isn't a Math class – we're talking about physical systems. An equation doesn't just come from nowhere. Give me context. Give me thoughts. Give me what ideas you are trying to use! You may earn a little bit more credit if you include this and – more importantly – if you give me TEXT in your answers, I can better help you figure out if you make a conceptual error. If all I have to grade are a bunch of equations right after each other in a row, it is difficult for me (or anyone else) to figure out what you were doing. I will always be happier if you include more words to describe your ideas/reasoning/thoughts. I know it takes a bit more work, but the more you show me about your lines of thought, the more effective I can be in helping you attack these problems. You don't necessarily have to write in complete sentences, but sometimes a word or two makes all the difference in helping me understand what you were trying to do. (Of course, you don't have to take this to extremes – if you divide both sides of an equation by 2, I can probably figure that out without any text..... when in doubt, include more words. It can't hurt.)

You may work with your classmates on the homework, but it is expected that your final solutions are your own. Collaborate/discuss/work out elements of the problem with each other if you wish – but when it comes time to write your actual solution, don't copy from each other – get the basic ideas of what you have to do and then go off and write your own answers.

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THIS homework is composed of problems (sometimes slightly modified) that were given to me when I was taking the course that we would call PHYS 111 here at C of C. I am sure that you know all of the Physics you would need to solve these problems already, but you may be rusty. Hopefully running through these problems will refresh your memory a bit. If it seems to easy, don't worry. This is meant as review.

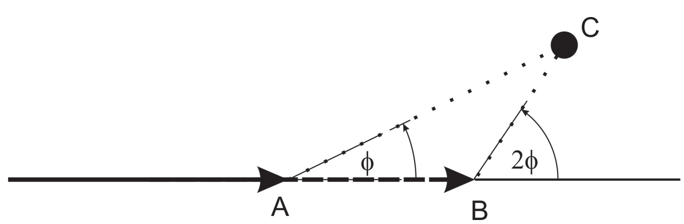
Note: Throughout the semester, you may assume all mechanical activities are taking place near the surface of Earth unless the context of the problem clearly suggests otherwise. You may assume Earth is spherical, and the value of $\vec{g}$ near the surface of the Earth is nominally 9.81 m/s^2, and points towards the center of the Earth. You may also assume all speeds are much less than the speed of light in a vacuum.

1. An automobile is speeding and moves steadily with speed v. The automobile passes a hidden stationary patrol car, which immediately takes off after it. Assume that the patrol car accelerates with constant acceleration a. (Not terribly realistic, but it is much easier to solve this way).

    a) How long will it take the patrol car to catch up with the speedster? (Assume all motion for both vehicles is along a straight line).

    b) What distance will the patrol car have covered when it catches up with the automobile?

2. By a method known as "doubling the angle on the bow," the navigator of a ship can determine their position relative to a fixed point, such as an island. The figure below will, hopefully, make the geometry a little more clear. The ship travels in a straight line (from A to B and beyond). Initially, at point A, an angle ϕ is measured between the direction of travel and an island. Later, at point B, the angle between the direction of travel and the island is 2ϕ. Show that the magnitude of vector AB is equal to the magnitude of vector BC. (This concept ends up being very useful in navigation).



3. This one is a deceptively tricky thought experiment. Suppose you tranquilize a polar bear on a frictionless glacier as part of a research effort. How can you estimate the mass of the bear using a measuring tape, a rope, and a knowledge of your own mass? In a few sentences, describe the procedure you would use, the measurements you would make, and show how the measured quantities enable you to determine the mass of the bear. Hints/reminders: remember that the surface of the glacier is presumably flat and – given the parameters of the problem – frictionless. Presumably you know (or can measure) the length of the rope. Also note that you don't have a stopwatch or any other reliable timing mechanism.

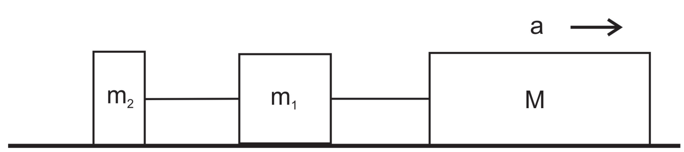
4. You might remember the "range formula" from PHYS 111 which states that, for a projectile launched at angle $\theta_\circ$ above the horizontal at initial launch speed $v_\circ$, the horizontal range over level land (in the absence of air resistance) is given by:

$$R(v_\circ, \theta_\circ) = \frac{v_\circ^2 \sin(2\theta_\circ)}{g}$$

Show that, for a particle that follows the maximum-range parabolic trajectory for a given initial speed (i.e. $\theta_\circ$ above is chosen to maximize R, assuming $v_\circ$ is held constant), the relation $H_{\text{max}} = \frac{R}{4}$ holds where H_{max} is the maximum height attained by the projectile and R is the range. (Again, you may neglect air resistance).

5. Three masses accelerate to the right as shown below. The rightmost mass has mass M, and the other two masses are m_1 and m_2 as shown. The three masses are connected by ropes of negligible mass. The masses slide on a frictionless surface. The rightmost mass accelerates with constant acceleration a. The entire force generating this motion is not shown, but acts directly on mass M only. (The other masses accelerate because they are pulled by M). Determine:

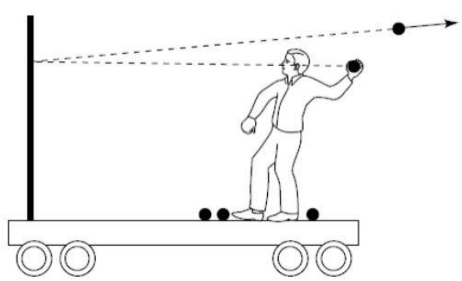
   a) The magnitude of the force applied to M to make this system move.
   b) The tension in the rope between M and m_1.
   c) The tension in the rope between m_1 and m_2.
   d) The net force on m_1.



(Remember, give all answers in terms of m_1, m_2, M, and a only!!!!)

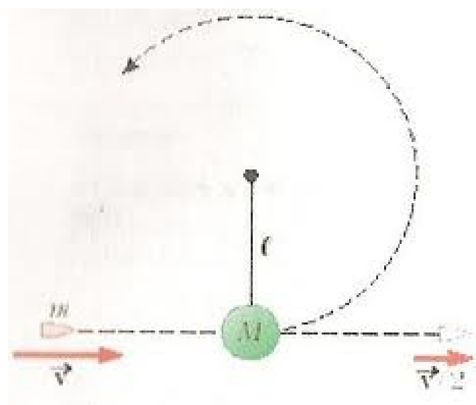
6. A spring toy consists of a piece of plastic attached to a spring. The spring is compressed Δx and the toy is released. After release, the entire spring launches some distance off of the surface. (It is no longer contacting the Earth). If the mass of the whole toy has mass M and its center of mass raises to a maximum height of h above its initial (uncompressed) center of mass, what is the force constant of the spring?

7. Suppose you are on a cart, initially at rest on a track with negligible friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back at you as shown in the figure below, is the cart put in motion? If so, which way? (To the left or to the right?)

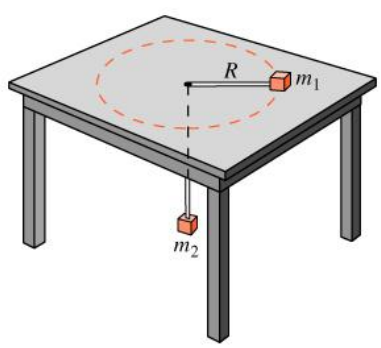


8. A satellite of mass m_1 orbits a body of mass m_2 in a circular orbit of radius R and period T. Use first principles to derive the relationship between radius R and period T. (Derive it – don't just quote the result. If necessary, you may assume $m_1 \ll m_2$ so that m_2 can be treated as effectively stationary).

9. A bullet of mass m and initial speed v passes completely through an initially stationary pendulum bob of mass M and length ℓ. The bullet emerges with a speed αv. (α is an unspecified constant somewhere between 0 and 1). The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle? (Note, the picture below assumes $\alpha = \frac{1}{2}$, but we want to keep the answer general here).



10. A block of mass m_1 slides in a circular path of radius R on a frictionless table. The mass is connected via an inextensible massless string to a mass m_2 hanging below the table. What is the speed of m_1?



11. Two astronauts, each with mass M, are connected by a rope of length d having negligible mass. They are isolated in space, orbiting their center of mass at speed v. Calculate:

   a) The magnitude of the angular momentum of the system. (Assume the astronauts are point particles).

   b) The rotational energy of the system.

   By pulling the rope, the astronauts shorten the distance between them to $\frac{d}{2}$.

   c) What is the new angular momentum of the system?

   d) What are their new speeds?

   e) What is the new rotational energy of the system?

   f) How much work is done by the astronauts in shortening the rope?

12. A block with mass M is released from height h above the level portion of the track shown below. The track is rough between points A and B, but elsewhere all surfaces are frictionless. As the block traverses the distance d between points A and B it loses mechanical energy E_1 ($E_1 < Mgh$). The spring constant of the spring affixed to the wall is k.

   a) Find the speed of the block at point A the first instant it passes through point A.

   b) Find the speed of the block at point B the first instant it passes through point B.

   c) What is the maximum compression of the spring during the motion of the block?

   d) What is the coefficient of kinetic friction between the block and the rough portion of the track?

   e) Assuming $E_1 < \frac{Mgh}{2}$, how high would the block reach on the first "return trip" up the triangular wedge?