Assignment II, PHYS 301 (Classical Mechanics)
Spring 2015
Due 1/23/15 at start of class

Important note: When you get this homework, we're probably in the process of starting projectile motion with air resistance. This homework is like PHYS 111 – we're assuming that everything occurs in a vacuum and that air resistance can be ignored. Your next homework (HW 3) will have stuff with air resistance.

1. You probably remember from PHYS 111 that the first derivative of the position vector is called the "velocity" and the second derivative of the position is known as the "acceleration". You may not know these terms, but the third derivative of the position is known as the "jerk". (e.g. the jerk is defined as ($\dddot{\vec{r}}$)). The fourth derivative of the position is known as the "jounce" or the "snap".

   a) What do the units of the snap have to be?

   b) The one-dimensional kinematic equation $x = x_\circ + v_\circ t + \frac{1}{2}at^2$ is based on the assumption of constant acceleration. If, instead, you had constant "snap" (call it s), what would the equivalent (one-dimensional) kinematic equation be? Assume the particle starts at position $x_\circ$ with velocity $v_\circ$, acceleration $a_\circ$, and jerk $j_\circ$.

   c) Show that the expression for jerk in polar coordinates is given by:

   $$\vec{j} = \left(\dddot{r} - 3r\dot{\phi}\ddot{\phi} - 3\dot{r}\dot{\phi}^2 \right) \hat{r} + \left(3\ddot{r}\dot{\phi} + 3\dot{r}\ddot{\phi} + r\dddot{\phi} - r\dot{\phi}^3 \right) \hat{\phi}$$

   d) Simplify the above expression for the case of uniform circular motion (where r and $\dot{\phi}$ are both constant). From this simplified expression, find a simple expression for the snap in polar coordinates for uniform circular motion. Remember, this is a vector! As a check, it should have the same units as your answer to part (a).

2. A ball is thrown straight upward (on Earth, from a level surface) so that it reaches maximal height H. The ball then falls down and bounces repeatedly. After each bounce, the ball returns to a certain fraction χ of its previous height.

   a) Find the total distance traveled (not displacement) by the ball.

   b) Find the total time until the ball stops moving.

   Note – both (a) and (b) above should give you a *finite* result. Both of these results converge.

   More on Back!!!

3. A ball is thrown with speed $v_\circ$ from the edge of a cliff (on Earth) having height H.

   a) How long until the ball hits the surface of the Earth as a function of inclination angle θ? (If there is any ambiguity, θ is measured at the top of the cliff and is the inclination from a horizontal line at the cliff's elevation).

   b) What horizontal distance does the ball travel as a function of inclination angle θ? (Assume that the ground below the cliff is horizontal)

   c) The angle that maximizes this horizontal distance can be found (after much painful algebra) to be: $\theta = \sin^{-1}\left[\left(\alpha + \frac{2gH}{v_\circ^2}\right)^{-1/2}\right]$. With α a numerical constant. Find α. (One way to do this would be to actually take the derivatives and do a LOT of simplification. I didn't do it that way, however, and I'd advise you only to go that route as an absolute last resort. Here's a hint – any H at all works here. You should know the result for one value of H from PHYS 111).

4. A charged ball is launched with speed $v_\circ$ and at angle θ above the horizontal, aimed to the right on a level surface (on Earth, with our normal gravity). This charged ball is exposed to a horizontal electric field pushing the ball further to the right with a (constant) force equal to the ball's weight.

   a) Under these conditions, what is the range equation for the projectile?

   b) What angle θ optimizes the range? I want an actual angle here! And no calculators or Mathematica! It works out pretty nicely.

5. At $t = 0$, a projectile is fired with speed $v_\circ$ at an angle θ above a horizontal surface. On this planet, gravity behaves really, really weird. In fact, here we have a time-dependent gravity that obeys the basic form $g = \alpha t^2$, so that it increases quadratically with time - starting with a value of 0 when the projectile is fired. (α is an undetermined constant).

   a) What do the units of α have to be?

   b) What horizontal distance does the projectile travel before landing?

   c) What value of θ should be chosen to maximize the range? I want an actual angle here! And no calculators or Mathematica! It works out really nicely.

6. Two cannonballs are launched (on Earth) from the same flat field with speeds v_1 and v_2 at angles (with respect to the horizontal) of φ_1 and φ_2, respectively. ($\varphi_1 > \varphi_2$). Show that if the two cannonballs are to collide in mid-air, the time-interval between firings must be:

$$\Delta t = \frac{2v_1 v_2 \sin(\varphi_1 - \varphi_2)}{g(v_1 \cos \varphi_1 + v_2 \cos \varphi_2)}$$

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