Assignment II, PHYS 301 (Classical Mechanics)
Spring 2017
Due 2/3/17 at start of class

As always, do not use technological aids to solve any of these problems unless I tell you that the use of computer algebra systems or calculators is ok.

Please turn in your clear, legible solutions on separate paper.

1. The position of a particle in mechanics is usually written as $\vec{r}$. Its velocity is written $\vec{v}$ (or $\dot{\vec{r}}$) and its acceleration is written $\vec{a}$ (or $\ddot{\vec{r}}$). The time derivative of the acceleration ($\dot{\vec{a}} = \ddot{\vec{v}} = \dddot{\vec{r}}$) is known as the "jerk" and the fourth derivative of position is known as the "jounce" or the "snap".
   (a) The one-dimensional kinematic equation $x = x_\circ + v_\circ t + \frac{1}{2}at^2$ assumes acceleration is constant. If, instead, you had constant "snap" (call it s), what would the equivalent (one-dimensional) kinematic equation be? Assume the particle starts (time $t = 0$) at position $x_\circ$ with velocity $v_\circ$, acceleration $a_\circ$, and jerk $j_\circ$.
   (b) Find an expression for jerk in polar coordinates. (Hint – you likely want to start with the expression for acceleration in polar coordinates and then carefully take a time derivative). Make sure to simplify your answer!
   (c) Simplify your answer to part (b) above under the case of uniform circular motion (where r and $\dot{\phi}$ are both constant, thus leaving $\dot{r}, \ddot{r}, \dddot{r}, \ddot{\phi}$, and $\dddot{\phi} = 0$. Remember your answer should be a vector!

2. A charged ball is launched with speed $v_\circ$, and at an angle θ above the horizontal, aimed to the right on a level surface (on Earth, with our normal gravity). This charged ball is exposed to a horizontal electric field pushing the ball further to the right with a (constant) force equal to the ball's weight.
   (a) Under these conditions, what is the range equation for the projectile? (In other words, what is the range R as a function of θ and $v_\circ$?)
   (b) What angle θ optimizes the range of this system? (No calculators or Mathematica! It works out pretty nicely).

3. Let mass m be constrained to move along the positive x-axis subject only to a velocity dependent drag. The drag takes the form:

$$\vec{F} = -\alpha v^4 \hat{v}$$

   (a) What do the SI units of α have to be?
   (b) Find an expression for $v(t)$ as a function of time t if the particle starts with initial velocity $v_\circ$. (For simplicity, assume $v_\circ > 0$ so that the particle starts by moving to the right.) (Remember – fractions within fractions are evil – leave your answer simplified enough so that they don't exist in your solution).
   (c) The quantities m, $v_\circ$, and α can be combined to give something with units of time, which will be the natural time τ for this system. (In other words, $\tau = \alpha^a v_\circ^b m^c$ for some constants a, b, and c so that τ has units of time). Find an expression for the "natural time" of this system.
   (d) Plug in τ for t in your answer to part (b) to find $v(\tau)$.

4. In class, Dr. Larsen derived the expression for v_y – the vertical component of the velocity for the motion of a projectile due to a linearly dependent drag force. The expression he obtained was:

$$v_y(t) = \exp\left[-bt/m\right]\left(v_{y\circ} - \frac{mg}{b}\right) + \frac{mg}{b}$$

with exp(mess) being a standard notation to indicate $\mathrm{e}^{\mathrm{mess}}$, and the expression describing the air drag as $\vec{f}_{\mathrm{drag}} = -b\vec{v}$.

   (a) As $t \to \infty$, $v_y \to v_t$ (where v_t is known as the terminal velocity). What is v_t?

   (b) If the object is dropped from rest (so $v_{y\circ} = 0$, how long would it take for something to reach speed αv_t with α some positive constant between 0 and 1?

   (c) Use the expression above for $v_y(t)$ to show that $y(t) = \frac{m}{b}\left\{\left[1 - \exp(-bt/m)\right]\left(v_{y\circ} - \frac{mg}{b}\right) + gt\right\} + y_\circ$

5. We talked about both linear and quadratic air resistance, but we never attempted to really do anything in the domain where both terms are non-negligible. Let $\vec{F}_d = -b\dot{\vec{y}} - c\dot{y}^2\hat{y}$ and we are trying to find the terminal velocity of an object of mass m in free-fall near earth. (Hence, the forces on the object are gravity and the two separate terms of the drag force).

   (a) What is the terminal velocity of the mass (in terms of b, m, c, and/or g)?

   (b) What is the approximate terminal velocity of the mass (in terms of b, m, c, and/or g) in the limit as $mg \ll \frac{b^2}{4c}$? (Note that you have to keep the first non-zero term of your answer; the correct answer is not zero).

6. Two astronauts, each of mass M, play catch with a football in space. The football has mass m and each astronaut always throws the football with speed v_{rel} *with respect to themselves.* Both astronauts start at rest, and initially astronaut A (on left in the figure as shown) is holding the football. You may neglect any gravitational attraction between the two astronauts. Give all answers with respect to an observer (astronaut "C") that is initially at rest with respect to both astronauts and remains in this same inertial reference frame throughout the problem. (i.e. give all answers with respect to the only initially relevant reference frame). Note that $v_{\text{rel}} \ll c$, so we don't have to worry about relativistic effects at all.

   a) Let "to the right" be indicated with a positive velocity. After astronaut "B" catches the football for the first time, what are the velocities of astronaut "A" and astronaut "B"? (As always, leave your answer in terms of the variables given in the problem statement only). Signs on this problem (all parts) are likely to get tricky, so leave your answer in terms of just positive variables and indicate if the final velocities are "to the left" or "to the right".

   b) After catching the football thrown by astronaut "A", "B" then throws the football back to astronaut "A". After astronaut "A" catches the football, what are the velocities of astronaut "A" and astronaut "B"? (Just for clarity in the next part of the problem, this is the status after 2 throws).

   c) What are the velocities of the astronauts after N throws? (This took me a surprisingly LONG time; eventually you hope to see a pattern, but it took me longer than I thought it would/should. Don't stress too much about part C if you can't get it; come back to it later after you finish the assignment). I left my answer in terms of v_{thrower} and v_{catcher} since they keep switching roles. (i.e. if we're talking the 4th throw, v_{thrower} corresponds to the most recent thrower, which would be astronaut B).

   d) At some point, the astronauts move away from each other at a speed that exceeds v_{rel}. At this point, they can't play catch anymore because a thrown ball will never reach the other astronaut. Figure out how many throws this takes. This is rather tricky, so I'll give you a giant hint; the answer takes the general form:

$$N > \frac{\ln(f(m, M))}{\ln(g(m, M))}$$

   where $f(m, M)$ and $g(m, M)$ are functions that you're working to find. If you have the right answer, you should find that for $m = 2.5$ and $M = 80$, the astronauts are able to complete 23 passes, but the 24th pass never catches up to the astronaut floating away.