Assignment II, PHYS 308 (Atmospheric Physics)
Fall 2016
Due 9/2/16 at start of class

NOTE: If you have had me for classes before, you know that normally I want you to leave answers symbolically. In *THIS* class, however, leave answers numerically if you can. (In other words, give me a number at the end of the problem). The reason is that this class is more directly applied to a particular subfield than most of the other upper-level classes you may have had from me (like Mechanics, E&M, or Modern) and, as such, we want to get you a sense of the magnitude of these things. Note – this also means you *will* be allowed to use calculators on exams.

Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't read it (or I can't follow your reasoning). Extensive exposition on your thought process or strategy is always appreciated. Don't just give me a string of equations! A solution involves a thought process/words as well as mathematical manipulation!

1. In class, I gave you a breakdown of the constituents of dry air (by volume) in the atmosphere. Note that, for gases, fraction "by volume" is the same as fraction "by number". In other words, when I say that about 21% of air is O_2, that means that about 21 out of every 100 air molecules is O_2. Since different molecules weigh different amounts, however, that does *not* mean that 21 kg out of every 100 kg of the atmosphere is composed of O_2.

   a) In class, we will frequently assume that the average gas molecule in the Earth's atmosphere weighs about $0.028\text{kg}/6.022 \times 10^{23} = 4.65 \times 10^{-26}$ kg. Use the real abundances of molecules (for dry air) to determine what mass we *should* use for an average gas molecule in the Earth's atmosphere. (For example, if air were made of 30% N_2 and 70% CO_2, the actual average gas molecule would weigh $(0.3)(0.028\text{kg}/6.022 \times 10^{23}) + (0.7)(0.048\text{kg}/6.022 \times 10^{23}) \approx 6.97 \times 10^{-26}$ kg.)

   b) Compare the value you calculated above in part (a) to the value of an average water molecule. If the total number of molecules per volume remains the same in moist air compared to dry air, which type of air would weigh more? Justify your answer with a computation.

   c) It is often said that moist air feels "heavy". Is this consistent with your computations above? If not, try to explain why people say that.

2. Calculate the total number of molecules in the Earth's atmosphere. You may have to look up the Earth's radius, and remember that standard sea-level pressure on Earth is 101325 Pa.

3. This problem deals with temperature conversion formulae. As a reminder, I give you the following:

$$T_K = T_C + 273.15$$
$$T_F = \frac{9}{5}T_C + 32$$
$$T_R = T_F + 469.67$$

   a) There is one (and only one) temperature where the Celcius and Fahrenheit scales agree. Find that temperature.
   b) Using the formulas above, derive an expression that gives the Kelvin temperature in terms of the Fahrenheit temperature. (E.g. your answer should be in the form $T_K = f(T_F)$).
   c) There is one (and only one) temperature where the Kelvin and Fahrenheit scales agree. Find that temperature.

4. In class, we derived the relationship:

$$P(z) = P_\circ e^{-z/H}$$

   where H is the scale height of the atmosphere. From this, you can also show that the number density of particles follows the similar relationship:

$$n(z) = n_\circ e^{-z/H}$$

   (for now, accept this without proof).

   a) Let's say that the density of air at sea level is 1.25 kg/m^3. What must $n_\circ$ be? (You may wish to use a result from earlier in this homework).
   b) Let's say that we can effectively say that the atmosphere ends when $n(z)$ drops below 1 molecule per cubic meter. If that's the case, how high would the atmosphere go? (For simplicity, ignore the variability of g (which is in the expression for H)).

5. It may seem strange to you that we asserted that atmospheric pressure at some height z depends only on the total weight of the air above height z. After all, the air above you isn't in contact with you, so how can a Nitrogen molecule 30 km above your head have direct influence on the pressure you feel? To help this make more sense, we'll look at the force associated with an ordinary mechanical collision (think back to PHYS 111)....

   Consider a perfectly elastic ball of mass m bouncing up and down on a horizontal surface under the action of a downward gravitational acceleration g. The ball is initially dropped from height h and always rebounds to this same height (since the ball is perfectly elastic). Calculate the average force applied to the horizontal surface by the ball.

   Hints/clues/observations: Most of the time – while the ball is in the air – there is no force applied on the surface by the ball. During the bounce, however, the ball will impart momentum to the surface. After n bounces, this momentum will have been applied to the surface n times. Recall the definition of a force is $\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}(= m\vec{a})$. If we interpret this for this context as $\vec{F} = \frac{\Delta p}{\Delta t}$ where Δp is the total momentum transferred to the table after n bounces and Δt is the time it takes to make n bounces, we have a rather interesting way to measure average force. Since I only gave you parameters m, g, and h it is reasonable to assume your final answer will only depend on these variables (or a subset of them) along with fundamental constants of nature.