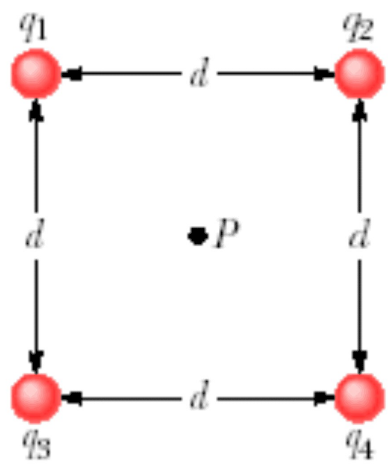
Assignment III, HONS 158 (Honors Physics II)
Spring 2016
Due 1/27/16 at start of class

As always, please put your answers on separate paper.

1. Two protons are held 1 cm apart. One of the protons is "released" (from rest) while the other is held and unable to move. What is the speed of the proton when it is very, very far away from the stationary proton? (E.g. what is the speed of the moving proton as the distance between the two charges goes to infinity?) (If the idea of using infinity bothers you, then find the speed when the protons are separated by 1×10^{10} m instead).

2. Examine the figure below, corresponding to a fixed charge configuration. (In other words, q_1, q_2, q_3, and q_4 are not able to move). In this problem, leave your answers in terms of Q, d, and fundamental constants.

   a) If $q_1 = q_2 = q_3 = q_4 = Q$, what is the magnitude of the electric field at point P?
   b) If $q_1 = q_2 = Q$ and $q_3 = q_4 = -Q$, what is the magnitude of the electric field at point P?
   c) If $q_1 = q_4 = Q$ and $q_2 = q_3 = -Q$, what is the magnitude of the electric field at point P?
   d) If $q_1 = q_2 = q_3 = Q$ and $q_4 = -Q$, what is the magnitude of the electric field at point P?



3. Let us define the following vectors:

$$\begin{aligned} \vec{t} &= 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k} \\ \vec{u} &= 5\hat{s} + (7\pi/6)\,\hat{\varphi} - 2\hat{z} \\ \vec{v} &= 3\hat{r} + (2\pi/3)\,\hat{\theta} + (4\pi/3)\,\hat{\varphi} \\ \vec{w} &= 4\hat{s} + 0\hat{\varphi} + 0\hat{z} \end{aligned}$$

You will be asked below to convert these vectors to vectors in different coordinate systems. Leave angles in radians, with θ taking on a value between 0 and π and φ taking on a value between 0 and 2π. (Numerically approximate answers to 3 significant figures).

a) Vector $\vec{t}$ is currently written in Cartesian coordinates. Convert it to Cylindrical coordinates.

b) Now convert vector $\vec{t}$ into Spherical coordinates.

c) Vector $\vec{u}$ is currently written in Cylindrical coordinates. Convert it to Cartesian coordinates.

d) Now convert vector $\vec{u}$ into Spherical coordinates.

e) Vector $\vec{v}$ is currently written in Spherical coordinates. Convert it to Cartesian coordinates.

f) Now convert vector $\vec{v}$ into Cylindrical coordinates.

g) Vector $\vec{w}$ is currently written in Cylindrical coordinates. Convert it to Cartesian coordinates.

h) Now convert vector $\vec{w}$ into Spherical coordinates.

4. Let:

$$\begin{aligned}
\vec{a} &= yz\hat{x} + xz\hat{y} + xy\hat{z} \\
\vec{b} &= x^2 e^{-z}\hat{x} + y^3 \ln(x)\hat{y} + z\cos(xyz)\hat{z} \\
\vec{c} &= r^2\hat{r} \qquad \text{(Spherical coordinates)} \\
\vec{d} &= \frac{1}{s^2}\hat{s} \qquad \text{(Cylindrical coordinates)} \\
f &= 3x^2yz \\
g &= \cos(xy) \\
h &= r^3 \qquad \text{(Spherical coordinates)}
\end{aligned}$$

Compute the following:

a) $\vec{\nabla} \cdot \vec{a}$

b) $\vec{\nabla} \cdot \vec{b}$

c) $\vec{\nabla} \cdot \vec{c}$

d) $\vec{\nabla} \cdot \vec{d}$

e) $\vec{\nabla} \times \vec{a}$

f) $\vec{\nabla} \times \vec{b}$

g) $\vec{\nabla} \times \vec{c}$

h) $\vec{\nabla} \times \vec{d}$

i) $\vec{\nabla} f$

j) $\vec{\nabla} g$

k) $\vec{\nabla} h$