Assignment III (Modified), PHYS 230 (Modern Physics)
Fall 2019 Due September 12th, 2019 at Start of Class

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

1. Your instructor skipped some steps in getting to the final Lorentz transformation (in the spherical light wave approach). Given his intermediate step:

$$x^2 - c^2t^2 = [\lambda(x - vt)]^2 - c^2\left[\frac{(1-\lambda^2)x}{\lambda v} + \lambda t\right]^2$$

equate coefficients of the t^2 term to show

$$\lambda = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

[hint, you're going to have to multiply out the right hand side above and then figure out what λ has to be so that the sum of all the coefficients multiplying the t^2 and add up to $-c^2$ like they do on the right hand side].

2. In class, we either have derived (or are in the process of deriving) the Lorentz Transformation. The final result of this transformation reads:

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2) \end{aligned}$$

With $\gamma \equiv \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$. For this problem, let v (the relative velocity between the primed and the unprimed frame) be equal to $3c/5$.

a) What is the value of γ for this relative speed between the frames?

b) If an event (E1) occurs at the coordinates $x = 3.1$ m, $y = 2.3$ m, $z = -3.4$ m, and $t = 10$ ns, what are the coordinates of the event E1 in the primed frame?

c) If a different event (E2) occurs at coordinates $x = -2.1$ m, $y = -7.1$ m $z = 0$ m, and $t = 7.5$ ns, what are the coordinates of event E2 in the primed frame? [You should notice that in the primed frame, event 2 occurs after event 1, even though in the unprimed frame event 1 occurs after event 2!]

d) If an event (E3) occurs at the origin of the primed frame at $t' = 10$ s, at what coordinates did the event occur in the unprimed frame?

3. Your task: Approximate each of these expressions for $v \ll c$. In your final answer, keep the first two non-zero terms for each answer.

   a) γ. If you forgot, $\left(\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$.

   b) γ^{-1}

   c) $1 - \frac{1}{\gamma}$

   d) $\frac{1}{1+\frac{v}{c}}$

   e) $\sqrt{\frac{2}{c^2-v^2} + \frac{2}{c^2+v^2}}$

4. The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of $0.35c$ with respect to Earth, determine the speed of the faster spaceship (with respect to Earth). Leave your answer as a decimal fraction of c (e.g. $v = 0.142c$). Leave your answer to at least 3 significant figures.

5. When at rest, the Σ^- particle has a lifetime of 0.15 ns before it decays into a neutron and a pion. One particular Σ^- particle is observed to travel 3.0 cm in the lab before decaying. What was its speed. Leave your answer as a fraction of c (e.g. in a form that looks like $\frac{3c}{4}$ [not the right answer]. (Hint: The correct answer is not $\frac{2}{3}c$).

6. A Physics professor on Earth gives an exam to his students who are on a spaceship traveling at speed v relative to the Earth. The moment the ship passes the professor, he signals the start of the exam. If he wishes his students to have time $T_\circ$ (spaceship time) to complete the exam, show that he should wait a time (Earth time) of:

$$T = T_\circ \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

   before sending a light signal telling them to stop. Hint: Remember that it takes some time for the second light signal to travel from the professor to the students.

   Note that I realize the irony that I just told you that fractions within fractions are evil and shouldn't be used. Thus, for self-consistency, I will note that the expression above can be written $T = T_\circ\sqrt{\frac{c-v}{c+v}}$.

7. Show that if $0 < v_1 < c$ and $0 < v_2 < c$ are two velocities pointing in the same direction, the relativistic sum of these velocities, v, is greater than $v_!$ and greater than v_2 but less than c. (This means prove it; don't just plug in one particular case).

8. If the total energy of a particle of mass m is equal to twice its rest energy, then what is the magnitude of the particle's relativistic momentum? (Leave your answer in terms of m, c, and numerical constants).

9. A particle of mass M decays from rest into two particles. One particle has mass m and the other particle is massless. In terms of M, m, and c, what is the momentum of the massless particle?

10. This problem is designed to help you get a sense of the scale of the energy "stored" in mass. For this problem I'm looking for numerical answers.

    a) Compute the rest energy of a paperclip (estimate or measure the mass somehow).

    b) If you convert this energy entirely to electrical energy and sell it for 8 cents per killowatt-hour (approximately the US national average), how much money would you get? (Assume you paid a penny for the paperclip and you found a way to convert the rest energy to electrical energy without any cost to you).

    c) If you could power a 60 Watt lightbulb with the energy, how long would the bulb stay lit?

11. An electron with rest energy $mc^2 = 0.511$ MeV moves with respect to the laboratory at speed $u = 0.95c$. Find (a) γ, (b) p (in units of MeV/c), (c) E, and (d) the (relativistic) kinetic energy of the electron. (I'm looking for numerical answers on this problem).

12. Show that, for a particle moving close to the speed of light, the particle speed v differs from the speed of light c by:

$$c - v \approx \frac{c}{2}\left(\frac{mc^2}{E}\right)^2$$

    in which E is the total relativistic energy.