Assignment III, PHYS 230 (Introduction to Modern Physics)
Spring 2017
Due 2/1/17 at start of class

As we transition to more sophisticated Physics classes, you will frequently be asked to leave your answers symbolically. Thus, you will have more answers that look like $5v/3$ than 2.78×10^7 m/s. This transition might be a bit confusing at first, so for the next couple of assignments, I will try to make it clear if your answer should be symbolic (like $5v/3$, where you don't include units and may have variables in your answer) or numerical (like 2.78×10^7 m/s, where you DO include units and the final answer should be an unambiguous quantity).

Note that when you do leave a symbolic answer, you should leave your answer in terms of variables specified in the problem statement (and obvious physical constants like $\vec{g}$, G, c, etc.) ONLY. This will become easier to understand with experience, but – for now – if you aren't sure if you can leave a particular quantity in your answer, please stop by and ask.

1. In deriving the Lorentz transformation, Dr. Larsen had the following expression:

$$x^2 - (ct)^2 = [\lambda(x - vt)]^2 - c^2 \left[\frac{(1-\lambda^2)x}{\lambda v} + \lambda t \right]^2$$

   In class, Dr. Larsen equated coefficients of the x^2 term on both sides of this equation to show that $\lambda = \gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. Here, I want you to do the same thing, except I want you to equate the coefficients of the t^2 term instead. (You should get the same result – that $\lambda = \gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.) (This is a symbolic problem).

2. The proper length of one spaceship is 5 times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of $0.4c$, determine the speed of the faster spaceship. (Leave your answer symbolically in terms of c, e.g. something like $\sqrt{\frac{83}{89}}c$ [though that isn't the right answer]).

3. There is a subatomic particle known as the Σ^- particle. When at rest, the Σ^- particle has a lifetime of 0.15 ns before it decays into a neutron and a pion. One Σ^- particle is observed to travel 3.0 cm in the lab before decaying. What was the particle's speed? (Hint: Its speed was NOT (2c/3)). (Leave your answer symbolically in terms of c (e.g. something like $\frac{2c}{\sqrt{5}}$, though that isn't the right answer. Assume $c = 3 \times 10^8$ m/s).

4. The mean lifetime of a muon in its rest frame is 2.2 μs. Let's say some event creates a whole lot of muons that travel together in a group. If these muons are all traveling at $0.9998c$ with respect to Earth, how far do they travel (as measured by someone on Earth) before 80% of the particles have decayed? (Leave your answer numerically, and make sure not to round too early!)

   Hint: You may or may not remember this, but for a radioactive system, you can write the expected number of particles that survive after time t as: $N(t) = N_\circ \exp(-t/\tau)$ where $N(t)$ is the number of surviving particles after time t, $N_\circ$ is the initial number of particles, τ is the mean lifetime, and $\exp(x)$ is a common notation that means the exact same thing as e^x. This hint doesn't have anything to do with the relativistic parts of things, however. You'll have to add relativity to the picture to answer this question.

5. A rocket-ship moves at steady speed v away from Earth. That rocket-ship shoots a missile at an asteroid ahead of its path; the speed of the missile with respect to the rocket-ship is $3v$. If the speed of the missile as measured from Earth is $\frac{7v}{2}$, what is v? (Leave your answer symbolically in terms of c, e.g. $v = c/\sqrt{3}$ or similar).

6. Dr. Larsen gives an exam to his Classical Mechanics class. The class is on a spaceship traveling at speed u relative to Earth. The moment the ship passes Dr. Larsen (standing on Earth), he signals the start of an exam. (Assume at this moment that the spaceship passes close enough to Dr. Larsen that it takes negligible time for the signal to travel to the ship). If Dr. Larsen wants his students to have time $T_\circ$ (spaceship elapsed time) to complete the exam, show that he should wait a time (measured on Earth) of:

$$T = T_\circ \sqrt{\frac{c-u}{c+u}}$$

   before sending a light signal telling the to stop. Hint: Remember that it takes some time for the second light signal to travel from Dr. Larsen to the students on the spaceship.