Assignment III, PHYS 272 (MAP)
Fall 2014
Due 9/5/14 at start of class

Note! Even though this is the last homework assignment you turn in before the first exam, there will be material on the first exam that comes after the content covered in this assignment. If you have any questions about what will be on the first exam, please contact Dr. Larsen and he'll be glad to clarify any confusion.

1. In case you've forgotten, the hyperbolic trig functions are defined as follows:

$$\sinh x \equiv \frac{e^x - e^{-x}}{2}$$
$$\cosh x \equiv \frac{e^x + e^{-x}}{2}$$

   a) Using the expansions for e^x and e^{-x}, develop a simple expression for the expansions for $\sinh(x)$ and $\cosh(x)$.
   b) Use your expressions for sinh and cosh from part (a) to approximate $\sinh(0.5)$ and $\cosh(0.5)$ to accuracy within 1%.
   c) From the definitions, show that $\cosh^2(x) - \sinh^2(x) = 1$

2. Identify all three (distinct) solutions to the equation $z^3 = 27i$. Write your answers in $x + iy$ form.

3. Simplify and write in polar ($Re^{i\theta}$) form: $\left(\sqrt{3} - i\right)^8$.

4. Simplify and write in standard ($x + iy$) form: $(4 + i)(2 - i)^2 + (2 + i)(3 + i)$.

5. Simplify, and write in standard form: $\frac{(2-4i)}{(1+2i)(2-3i)}$.

6. Write *all* values of:
   a) $\ln(e^{-2})$
   b) $\ln(-3)$

7. There is a link between complex numbers and 2×2 matrices. For a complex number in the form $x + iy$, you can cast the same basic information in a purely *real* matrix of the form:

$$\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$$

a) How would you write the complex number $z = Re^{i\theta}$ in matrix form?

b) What is the determinant of the matrix you developed in part (a)?

c) How would you write the complex number $z^* = Re^{-i\theta}$ in matrix form?

d) Find $[z, z^*]$ (the commutator of the matrix representation of z with its conjugate. (To do this, merely calculate $zz^* - z^*z$).

8. *By hand* (no computers), solve the following system of linear equations via Gaussian elimination:

$$\begin{aligned} 2x + 5y + z &= 2 \\ x + y + 2z &= 1 \\ x + 5z &= 3 \end{aligned}$$

9. Define the following matrices as follows:

$$A = \begin{pmatrix} -1 & 3 \\ 2 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & -6 \\ -1 & 7 \end{pmatrix} \qquad C = \begin{pmatrix} y^2 & z^4 \\ x^2y^2 & yz \end{pmatrix}$$

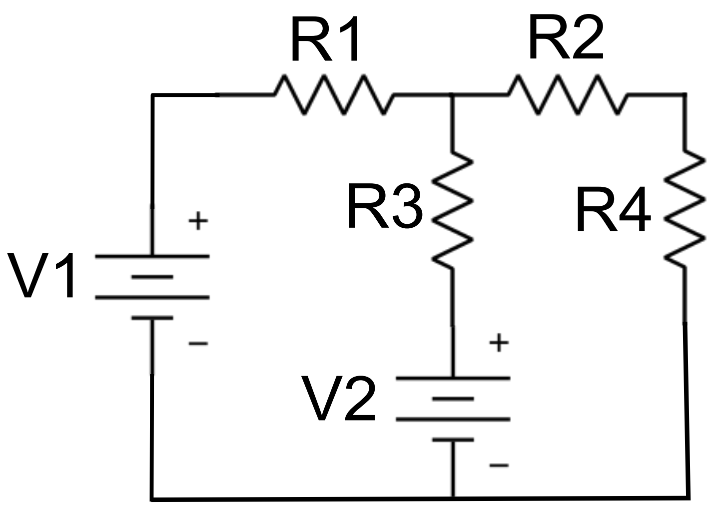
a) Find [A,A]

b) Find [A,B]

c) Find [B,A]

d) Find [B,C]

10. Examine the circuit below.



a) Find the current through $R2$. (Hint – there's a reason we're doing this right after we talked about row reduction. You may need to get a refresher on the use of Kirchhoff's Circuit Laws) Hint; you don't necessarily need to know about the current everywhere to get the current through $R3$. Even just finding the one current, it gets kinda messy.

b) Let $V_1 = V_2 = V$, and let $R_1 = R_2 = R_3 = R_4 = R$. Simplify your answer in part (a) to show that, in this case, the current through R_3 is given by $|I| = \frac{V}{5R}$.