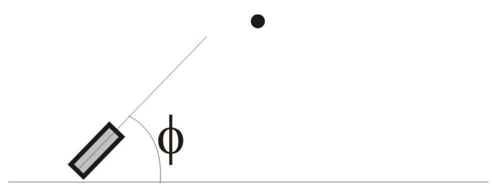
Assignment III, PHYS 301 (Classical Mechanics)
Spring 2014
Due 1/24/14 at start of class

On all homework assignments, only use a computer algebra system (e.g. *Mathematica*) if you are specified to do so. Feel free to *check* your work with a computer algebra system anytime, but I want to see you work through the math unless specified otherwise.

1. In class, we developed equations for velocity and acceleration in polar coordinates. Let's take this one step further.
   a) Find an expression for jerk ($\dddot{\vec{r}}$) in polar coordinates.
   b) Simplify the above expression for uniform circular motion with r and $\dot{\phi}$ constant.

2. A projectile is fired over a level surface with a speed $v_\circ$ such that it passes through two points both a distance h above the horizontal. (The first time height h is reached is on the projectile's ascent; the second time on its descent).



   a) Neglect the effect of any air resistance. Show that if the gun is adjusted for maximum range, the horizontal distance the projectile travels between these two points is equal to:

$$\frac{v_\circ}{g}\sqrt{v_\circ^2 - 4gh}$$

   b) Again, neglect the effect of any air resistance. Find an expression for the time of flight for the projectile between the points a distance h above the horizontal if the initial launch angle $\phi = \pi/6$. Your answer should be in terms of variables $v_\circ$, h, and g *only*.
   c) In your answer to part (b), you should get a nonsensical result (an imaginary time interval) when $h > \frac{v_\circ^2}{8g}$. Explain why.

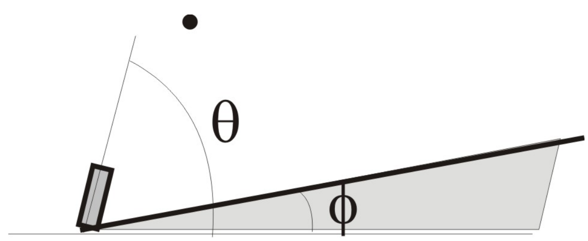
3. Let a projectile be launched from the surface over level terrain on Earth. Find an expression for the maximum height of the projectile in terms of total time-of-flight (t_{tot}), g, and numerical constants only. (Neglect air resistance). [Note – isn't it strange that this is true no matter what $\theta_\circ$ and $v_\circ$ are?!]

4. A cannon launches a cannonball with initial angle θ and velocity v up a hill having an inclination angle of ϕ. The range of the gun measured up the slope of the hill (if we neglect air resistance) is:

$$\frac{2v_\circ^2 \cos\theta \sin(\theta - \phi)}{g\cos^2\phi}$$

(You don't have to derive this. Consider it a beginning of the semester gift). Show that the maximum value of the range of the gun measured up the slope is

$$\frac{v_\circ^2}{g(1+\sin\phi)}$$



5. Let mass m be constrained to move along the x-axis subject only to a velocity dependent force:

$$F = -F_\circ e^{v/V}$$

where $F_\circ$ and V are constants.

a) Find $v(t)$ if the initial velocity is $v_\circ > 0$ at time $t = 0$.
b) At what time is the mass instantaneously at rest?
c) Find an approximate expression (in terms of $v_\circ$, m, and $F_\circ$ only) for your answer to part (b) when $v_\circ \ll V$.

6. Let us say that a mass is constrained to move on the x-axis subject to two forces: (i) a force that opposes the motion linearly (linear drag) written $\vec{f} = -b\vec{v}$, and (ii) a constant frictional force $\vec{f} = -f_\circ \hat{x}$. (This isn't completely realistic, because this frictional force doesn't "turn off" once the particle is stopped like friction normally does). If the mass initially moves with velocity $v_\circ > 0$, find:

   a) An expression for $\vec{v}(t)$. (It will be a little messy, but not absurdly ugly).

   b) Find the time at which $v(t) = 0$.

   c) Your answer should go kinda crazy when $b = 0$, even though if you think about it, that's a reasonably friendly situation – no drag except for a constant friction force. That's just a PHYS 111 problem (motion under constant acceleration). Why does your expression in part (b) get weird? How could you fix it? (Note: the answer isn't the whole "friction turns off when the object stops moving" issue alluded to in the question; you should still be able to find the instantaneous moment when the object first stops).

7. In the year 2000, the powers that be officially changed the size of a ping-pong ball from 38 mm to 40 mm in an attempt to slow the game down a little. The mass of a ping-pong ball is approximately 2.7 grams.

   A good way to determine whether or not quadratic or linear drag is most appropriate is to calculate the Reynolds number, which can be found via:

$$Re = \frac{\rho L v}{\mu}$$

   With ρ and μ being properties of the fluid the particle is traveling through, and L and v being properties of the particle and its motion with respect to the fluid. For a spherical particle moving in air, using the diameter of the particle for L and the velocity of the particle for v is appropriate. If the Reynolds number is much larger than 1, a quadratic drag should be used; for a Reynolds number smaller than 1, a linear drag should be used. (Note that this is approximate!!!)

   Let us follow the book's suggestion and use $b = 3\pi\mu D$ for linear drag $\vec{f}_{\text{drag}} = -b\vec{v}$. Also, let's use $c = \gamma D^2$ with $\gamma = 0.25\text{N} \cdot \text{s}^2/\text{m}^4$ for quadratic drag $\vec{f}_{\text{drag}} = -cv^2\hat{v}$. Use $\mu = 1.8 \times 10^{-5}$ kg/(m s) for the dynamic viscosity of air.

   The text gives an expression for the terminal velocity in terms of the two cases in equations 2.26 and 2.53.

   (See back of page for the "question" part of this question)

a) Assuming linear drag, find the terminal velocity of a 40 mm ping-pong ball.

b) Assuming quadratic drag, find the terminal velocity of a 40 mm ping-pong ball.

c) By calculating the Reynolds number for the two cases above, suggest which of these two terminal velocities is more likely closer to the truth.

d) Given your answer to part (c), what is the ratio $v_{\text{term}}(40\text{mm})/v_{\text{term}}(38\text{mm})$? (this indicates the relative speed of the game with the new ball compared to the old ball).

8. Do problem 2.20 in your textbook; you may use MATLAB, *Mathematica*, Maple, Mathcad, Excel/Open Office (if you hate me), or LabView (if you hate yourself) to do this. Use the book's suggested values for the parameters. Print out your solutions and attach them to the end of your homework. I want to see your code in addition to your actual plots! Make sure to label your axes! If you prefer, you can use equation 2.37 instead of equation 2.36.