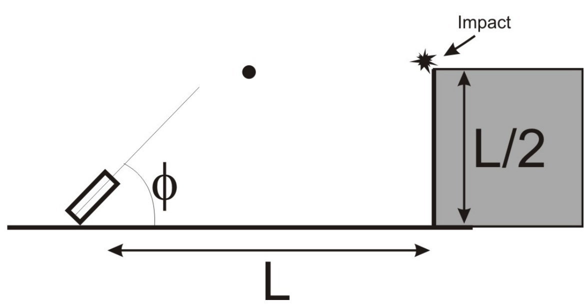
Assignment III, PHYS 301 (Classical Mechanics)
Spring 2015
Due 1/30/15 at start of class

1. There is nothing overly fancy about this problem; it is just a warm-up 111-on-steroids type problem to get you going. It is also the only problem on this homework set where you can assume the motion is happening in a vacuum (e.g. no air resistance).

   A cannon is being shot towards a cliff that lies a horizontal distance L away and rises to a height $\frac{L}{2}$ above the cannon barrel. The angle of elevation made with respect to the horizontal by the cannon barrel is ϕ.

   a) If the cannonball hits the very corner of the cliff, find the launch velocity in terms of L, ϕ, and fundamental constants.

   b) For certain angles, you should find that your answer to part (a) gives you an imaginary result. Explain why.



2. A spherical ball with radius R and (uniform) mass density ρ_{ball} falls vertically, due to gravity, in a fluid. The fluid has (uniform) mass density ρ_{f} and viscosity μ.

   a) Assuming that Stokes' flow is appropriate, what is the terminal velocity of the ball in the fluid? (Note – there are more than 2 forces here. In addition to gravity and drag, there's also a Buoyancy force that you'll need to take into account.)

   b) Let's say that $R = 1$ cm, the ball is made out of steel so $\rho_{\text{ball}} \sim 8000$ kg/m^3, and the fluid is glycerine so its density is $\rho_{\text{f}} \sim 1260$ kg/m^3 and $\mu \sim 1.4$ kg/(m s). Apply these values to your answer to part (a) to determine the terminal velocity of a steel ball in glycerine. (You may use a calculator or a computer algebra system to come up with this; I'm looking for an actual number – with units – instead of a symbolic answer here).

   c) Given your answer to part (b), was the assumption of Stokes' flow in part (a) appropriate? Justify your answer with a calculation. (Again, you may use a calculator or a computer algebra system to evaluate your expression to help answer the question).

3. Let mass m be constrained to move along the positive x-axis subject only to a velocity dependent drag. The drag takes the form:

$$\vec{F} = -\alpha v^3 \hat{v}$$

a) What do the units of α have to be in the SI system?

b) Find an expression for $v(t)$ as a function of time t if the particle starts with initial velocity $v_\circ$. (For simplicity, assume $v_\circ > 0$ so that initially the particle is moving to the right.) (Remember – fractions within fractions are evil. Leave your answer simplified enough so that they don't exist in your solution).

c) The quantities m, $v_\circ$, and α can be combined to give something with units of time, which will be the natural time τ for this system. (In other words, $\tau = \alpha^a v_\circ^b m^c$ for some constant a, b, and c so that τ has units of time). Find an expression for the "natural time" of this system.

d) Plug in τ in for t in your answer to part (b) to find $v(\tau)$.

4. A bomb is dropped from an airplane flying horizontally at height H with speed V. Assuming air resistance takes a linear form $\vec{f}_f = -mgC\vec{v}$, show:

a) The time of fall T is given exactly by the transcendental equation:

$$H = \frac{1}{gC^2}\left(e^{-gCT} - 1\right) + \frac{T}{C}$$

b) If you apply the assumption that the force of air resistance is small, show that the time of fall can be approximated by:

$$T \sim \sqrt{\frac{2H}{g}}\left(1 + \frac{1}{6}gC\sqrt{\frac{2H}{g}}\right)$$

Hint: Expand the exact expression to form a power series in T, then use the method of successive approximations to obtain the desired result.

c) Show the horizontal distance traversed by the bomb in its fall is approximately given by:

$$\Delta x \sim V\sqrt{\frac{2H}{g}}\left(1 - \frac{1}{3}gC\sqrt{\frac{2H}{g}}\right)$$

5. Dr. Larsen also occasionally teaches a first year class called “Physics of Sports”. In that class, we have tried to develop an experimental method to estimate the terminal velocity of a ping-pong ball. Here, I want you to come up with a strategy to do the same thing. Some extra information that will probably be helpful: you may assume that air resistance is negligible for a ping-pong ball dropped from a small, but still measurable heights (e.g. air resistance is negligible for, say, fall heights less than a half a meter or so). Although the bounce is not perfectly elastic, we will assume that the fraction of energy lost from collision with the floor is approximately constant. Note that you do not have access to a reliable timer, so direct measurement of velocities is not an option. All you have is the ping pong ball, a reliable measuring tape (so you can measure both drop and bounce heights for any initial drop height), and the ability to drop from any height that you wish. (We were actually able to do this back when we were in Bruner and came up with values that were within 10% of the generally accepted value). Energy considerations may be helpful, as might be the kinematic equations. Again – the question is to describe a strategy to experimentally determine the terminal velocity of a ping-pong ball subject to the constraints and assumptions outlined above. Be explicit enough to determine that you have a qualitative and quantitative understanding of the experimental design and inference of speed from measured variables.

6. A spherical ball of uniform density ρ_p is dropped from height h above a flat surface. In the absence of air resistance, you've done this problem plenty of times – you know that the relationship between fall-time and height can be written as $h = \frac{1}{2}gt^2$. Now, we're going to add a little bit of air-resistance and see what changes.

   a) Assuming a spherical ball with mass m is dropped from rest at a height h above a level surface, write an expression for the velocity as a function of time. We'll write the air resistance as $\vec{f_f} = -b\vec{v}$. Setting up the diffeq here is a bit tricky, but if you define $y > 0$ in the upward direction, you should start with the differential equation $m\dot{v_y} = -mg - bv_y$. Leave your answer in terms of m,g, b, and t. (You shouldn't have a v_{oy} since the ball was dropped from rest.

   b) From your expression for $v_y(t)$ above, it is possible to find an expression that links the height of the drop h to the time it takes to hit the ground. At this point, I'm not asking you to find this expression. (That's part (c) below). For now, we'll just mention that the answer is of the form $h = \frac{1}{2}gt^2 + \alpha t^3$ where α is some combination of constants related to the problem. (In the case of no air resistance, α is clearly 0). Given that h takes this form, what do the units of α have to be?

   c) Use your answer to part (a) to find an expression for α as described in part (b). Initially, you'll get a transcendental equation. Expand the exponential in a power-series to reach the desired form. You may assume bt/m is small for all values of t relevant to the problem.

   d) Recall that – for a uniform, solid sphere – we know that b can be written in terms of physically meaningful quantities related to the particle and fluid properties. Rewrite the expression $h = \frac{1}{2}gt^2 + \alpha t^3$ in terms of physically meaningful variables. Your answer should be in terms of D_p, ρ_p, ρ_f, μ, and/or fundamental constants. (Not all of these variables will necessarily come into play in your expression for α).

   e) Two balls are dropped from the same height H, both subjected to linear drag only. Ball 1 has diameter D_1, has density ρ_{p1}, and falls through a fluid with density ρ_{f1} and viscosity μ_1, Ball 2 has diameter $2D_1$, has density $\frac{1}{2}\rho_{p1}$, and falls through a fluid with density $3\rho_{f1}$ and viscosity $\frac{3}{2}\mu_1$. If both balls are dropped simultaneously, which one hits the ground first?