Homework 3, PHYS 415 (Fluid Mechanics)
Spring 2019
Due Thursday 24th January 2019 at Beginning of Class

As always, turn in your legible and annotated work on separate paper.

1. There is a point to this problem, but it may not be obvious at first blush. Take a spherical ball of mass m and radius R and drop it (from rest, near the surface of the Earth) with the bottom of the ball height H above a spring scale. This spring scale is flush with the ground and is infinitely responsive; rather than just reporting back a weight at random intervals, it actually outputs applied force as a function of time on an oscilloscope or strip chart or some other permanent record – giving you a means of knowing $f(t)$, the applied force as a function of t. Assume that the ball bounces without any energy loss (so it returns to height H) and keeps bouncing indefinitely. When H is small, the spring scale applies a modest force at regular intervals. When H is larger, the spring scale must apply a larger force, but less frequently.

   a) Find the average reading of the scale as a function of initial drop height H. (In other words, if the scale applies force $f(t)$, your task is to find $\langle f(t) \rangle$ as a function of H). Show your work to justify your answer.

   b) A microphysical picture of pressure isn't too dissimilar from the scenario outlined above. You have particles (e.g. gas molecules) bombarding a surface. Some molecules hit the surface with higher speed than others (after all not all gas molecules move at the same speed), and the collisions are treated as largely elastic. What does your answer to part (a) imply about the average force applied by a collection of molecules to a surface in a gravitational field (e.g. air molecules on the surface of the earth)?

2. The Reynolds number can be written as $\frac{\rho L u}{\mu}$ and the Froude number can be written as $\frac{u}{\sqrt{gL}}$. You have a system where a water flows at characteristic velocity 15 m/s in a geometry near the surface of the Earth with characteristic length scale 0.3 m.

   a) If you wanted to replicate the flow with glycerine in such a way as to have the same Reynolds and Froude number, what characteristic length and characteristic velocity would the glycerine flow have to have? (You may need to look up some constants for water and/or glycerine; assume 20° C and 1 atmosphere).

b) What would the characteristic length and characteristic velocity have to be if you wanted to replicate the flow with air instead? (You may need to look up some constants for air).

3. The Knudsen number ($\text{Kn} = \frac{\lambda}{L}$) compares the mean free path within a fluid (λ) to a characteristic distance L. When $\text{Kn} \ll 1$, the continuum assumption is valid. When $\text{Kn} \gtrsim 1$, then we need to abandon the continuum approach in favor of statistical methods.

   a) If we take L to be the size of an atmospheric particle, how small can a particle be to stay safely within the continuum assumption? (The mean free path for a non-interacting gas is $\frac{k_B T}{\sqrt{2}\pi D^2 P}$ where k_B is Boltzmann's constant, T the Kelvin temperature (use a reasonable surface air temperature), D the approximate size of the gas molecule, and P the (surface) pressure.)

   b) Assume that the mean-free path in the liquid is nominally the distance between molecules. Within this assumption, how small could a submerged particle be while still staying safely within the continuum assumption in water?

4. In class we have (or will) go through Buckingham's Π theorem. This allows us to construct dimensionless quantities out of combinations of variables (and/or constants).

   a) How many ways does Buckingham's Π theorem indicate you could combine e (the charge on an electron), $\epsilon_\circ$ (permittivity of free space), h (Planck's constant), and c (speed of light) into a dimensionless combination?

   b) Find all possible combinations of these variables that are dimensionless. [Note – no need to get cute. The Mach number is a dimensionless constant involving the speed of a fluid and the speed of sound: $M = v/c$. Technically, v^2/c^2 or c/v are also different dimensionless constants involving these same two variables, but I don't want an infinite list in your answer to this problem. If M is a dimensionless variable, of COURSE M^2 and $1/M$ are dimensionless variables as well. I only want "non-trivial" different combinations].

   c) Repeat parts (a) and (b) above, except for h (Planck's Constant), G (the universal gravitational constant), c (speed of light), and $a_\circ$ (the Bohr radius).

   d) Repeat parts (a) and (b) above, except now we'll use variables and/or material constants instead of universal constants: μ (dynamic viscosity), P (pressure), ρ (density), u (velocity), and D (diffusion coefficient).