Assignment IV, PHYS 308
Fall 2014
Due 9/26/14 at start of class

> NOTE: Just like last homework, please leave your answers in terms of actual numbers (with appropriate units) when possible. Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't read it (or I can't follow your reasoning). Extensive exposition on your thought process or strategy is always appreciated.

I have mentioned a number of times in class that showing the frequency distribution of atmospheric particulates as a function of size (e.g. a "size-distribution") can be deceiving. This is because many atmospheric processes depend on looking at variables besides particle number. It is often more important to consider the total surface area as a function of size, or total volume as a function of size. This problem is designed to start getting you to think about this.

1. Let us start by thinking about what happens to the total surface area when you divide a given volume of water among spherical drops of increasingly smaller size.

   a) If you have a single spherical drop of volume $V_\circ$, what is its surface area? (Please simplify your answer).

   b) If you divide the volume $V_\circ$ into n spherical drops, each of volume $V_\circ/n$, the total surface area of the collection of spherical drops can be written in the form $f(n) \cdot$ S.A. where S.A. is the answer to part (a). Find $f(n)$.

   c) The radar reflectivity factor can be written as $Z = \sum_i \kappa D_i^6$ with i summing over all of the drops in question, κ some (for now unspecified) constant and D_i corresponding to the drop radius of the ith drop. If you again divide the liquid volume $V_\circ$ into n spherical drops, each of volume $V_\circ/n$, find an expression for $Z(n)$.

   d) Let us push the previous scenario just a little bit further. Let us say that half of the volume ($V_\circ/2$) is separated into n_1 equal volume drops. The other half of the volume is separated into n_2 equal volume drops. Find an expression for $Z(n_1, n_2)$.

   e) In Physics, we often are concerned with obvious limiting cases. Compare the expression you developed in part (c) above to the expression you developed in part (d) above. To do this, define $N = n = n_1 + n_2$. In that case, if you've done parts (c) and (d) properly, you should see that $Z(N/2, N/2)$ (your answer to part (d) with both n_1 and $n_2 = N/2$) is equal to $Z(N)$ (your answer to part (c) with $n = N$). Verify that this works, and rewrite your answer to part (d) as $Z(n, N-n)$.

   f) Take the derivative of the expression you derived in part (e) above with respect to n. You should find that you hit an extremum when $n = N/2$. Verify this.

   g) Does the extremum of $Z(n, N-n)$ found when $n = N/2$ in the previous problem correspond to a min or max for $Z(n, N-n)$?

   h) Interpret your answer to part (g). What does this mean physically?

2. An average raindrop is 1 mm in diameter. An average cloud drop is about 10 μm in diameter. How many cloud drops does it take to make a raindrop?

3. Would you expect a fish swimming through water to have Kn $>$ 1 or Kn $<$ 1? Why? (You don't need to do a computation to answer this, but if you want to that is fine. Note, however, that liquid water isn't a gas – so the statistical $(3/2)kT$ argument used in class isn't appropriate here.

4. The maximum legal concentration of $PM_{2.5}$ particles in air is 15 micrograms per cubic meter of air. ($PM_{2.5}$ particles are particulate matter particles that are at least 2.5 micrometers in diameter). (Note – the maximum value presented above is a yearly average maximum; the instantaneous value is allowed to be higher than this). For simplicity, assume all $PM_{2.5}$ particles are 2.5 micrometers in diameter and also assume they are all made of liquid water. (These simplifying assumptions are obviously false, but our goal is to get an approximate value here). What is the maximum *number* concentration of $PM_{2.5}$ particles in the atmosphere in each cubic centimeter? (i.e. how many particles per cubic centimeter of air is the same as 15 micrograms per meter cubed if all the particles are spherical and have a diameter of 2.5 micrometers and the density of water (1g/cc)?)

5. Let us say that the Knudsen number associated with a particle near the surface is Kn_1. Would the same particle moving at the same speed in air 10 km above the surface be above Kn_1, below Kn_1, the same, or is it impossible to tell? Explain.

6. The terminal velocity of a settling 100 nm particle (due to gravity alone) is approximately 300 nm/second.

   a) What is the Reynolds number for this flow pattern? (You may assume that we're near the surface of the Earth and the particle is settling in air).

   b) Is this a turbulent flow?

7. A 5 cm diameter spherical stone of density 1200 kg/m^3 is thrown into liquid water.

   a) Assuming Stokes' flow is appropriate (it may or may not be), what is the terminal velocity of the stone? (Don't neglect buoyancy!!!)

   b) What is the Reynolds number for the flow as described in part (a)?

   c) Given your answer to part (b), was it appropriate to assume Stokes' flow? Why or why not?