Assignment IV, PHYS 230 (Introduction to Modern Physics)
Fall 2015
Due 9/17/15 at start of class

As always, please put your answers on separate paper.

1. You point a laser flashlight at the Moon, producing a spot on the Moon's surface.
   a) At what minimum angular speed ω must you sweep the laser beam in order for the light spot to move across the surface of the Moon with speed $v > c$. (For simplicity, you may assume the Moon is a flat disk and not a sphere).
   b) Why couldn't you use this method to transmit information between two different locations on the moon with a super-luminal (faster than light) velocity?
2. A thin rod of proper length $4a$ is traveling along the x-axis of a frame S with a speed $\sqrt{3}c/2$ in the positive x-direction. A hollow cylinder CD of proper length $2a$ is placed with its axis along the x-axis, so that when the ends of the cylinder are open the rod will pass through the cylinder. The end C of the cylinder is located at $x = -2a$ and the end D at $x = 0$, and both ends are equipped with hypothetical devices capable of closing them off with impenetrable and immovable walls.
   a) Show that in the frame S in which the cylinder is at rest, both the rod and the cylinder have length $2a$.
   b) Show that in the frame S' in which the rod is at rest, the length of the rod is $4a$, whereas the length of the cylinder is only a.
   c) Suppose at time $t = 0$ in frame S the front end B of the rod is located at $x = 0$, and both ends of the cylinder are suddenly closed. Is the rod trapped in the cylinder as might be inferred from the answer to part (a), or is the rod cut in two parts as might be inferred from the answer to part (b)? Reconcile this paradox.
3. How fast must you be moving toward a red light ($\lambda = 650$ nm) for it to appear:
   a) yellow ($\lambda = 590$ nm)?
   b) green ($\lambda = 525$ nm)?
   c) blue ($\lambda = 460$ nm)?
4. Show that the mass of a particle m is related to its kinetic energy K and momentum p according to the relation:

$$m = \frac{(pc)^2 - K^2}{2Kc^2}$$

MORE PROBLEMS ON BACK!!!!

5. Show that, for an extremely relativistic particle, the particle speed v differs from the speed of light c by:

$$c - v \approx \frac{c}{2}\left(\frac{mc^2}{E}\right)^2$$

in which E is the total energy.

6. An electron with rest energy $mc^2 = 0.511$ MeV moves with respect to the laboratory at speed $u = 0.95c$.

   a) What is γ between the electron and lab frames?

   b) What is p of the electron (in units of MeV/c)?

   c) What is the total energy E of the electron?

   d) What fraction of the electron's total energy is its Kinetic Energy?

7. a) Compute the rest energy of a paperclip (estimate or measure the mass somehow).

   b) If you convert this energy entirely to electrical energy and sell it to your friends and neighbors 9.74 cents per kiloWatt-hour (approximately the average price in South Carolina in June of 2015), how much money would you get? (Assume you paid a penny for the paperclip and you found a way to convert the rest energy to electrical energy without any cost to you).

   c) If you could power a 60 Watt lightbulb with the energy from the paperclip, how long would the bulb stay lit?

8. A positive kaon (K^+) has a rest mass of 494 MeV/c^2, whereas a proton has a rest mass of 938 MeV/c^2. If a kaon has a total energy that is equal to the proton rest energy, what is the speed of the kaon?

9. A particle moves with speed v at an angle θ with the x-axis in the S frame. Verify that its speed and direction in the S' frame are as given below (assume that the S' frame is moving with velocity u with respect to the S frame):

$$v' = \frac{\left\{v^2 - 2uv\cos\theta + u^2 - \left[\frac{uv\sin\theta}{c}\right]^2\right\}^{1/2}}{1 - \frac{uv\cos\theta}{c^2}}$$

$$\tan\theta' = \frac{\left(\sqrt{1 - \frac{u^2}{c^2}}\right) v\sin\theta}{v\cos\theta - u}$$