Assignment IV (expanded), PHYS 230 (Modern Physics)
Fall 2019 Due Tuesday October 8th, 2019 at Start of Class

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

Problems 1-4 of this homework assignment are to be done in MATLAB. Your answers to these problems should be sent to me via email to LarsenML@gmail.com and included in m-files named `YourLastName_Hw4Nr1.m`, `YourLastName_Hw4Nr2.m`, etc. Your m-file should be written in such a way that I just have to run the code to get the graphic and/or numerical result I'm looking for. (Note that I am having you send your answers to my personal email instead of my university email because CofC seems to strip-out .m files; you may need to email it to me from your personal email account. Make sure to put "PHYS 230 HW04" as your subject line in your email so I know it is legit.)

1. We just finished an extended section on relativity. Use MATLAB to make a nice, publication-quality plot of γ as a function of v/c. Make sure your m-file has code to label your axes. Make the y-axis logarithmic.
2. Same as the previous problem, but this time make the y-axis linear and go from 0 to 100.
3. As you know, we can approximate γ for small v/c with the expression $\gamma \approx 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4}$. Make a single plot where the x axis displays v/c in the range from 0 to 1 and the y-axis has the exact solution of γ in black, the two-term approximation $\gamma \approx 1 + \frac{v^2}{2c^2}$ in red, and the three term approximation $\gamma \approx 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4}$ in blue (all on logarithmic y-axes). Make sure your axes are labeled and that your figure has a legend.
4. Someone takes the following data for the photoelectric effect:

Incident λ (nm)	Stopping Voltage (V)
252.0	2.61
312.3	1.69
368.5	1.06
405.1	0.72
436.2	0.51

   a) Graph the data and use a linear fit in MATLAB to determine the work function for Lithium. (Hint – look up polyfit)
   b) Find the experimental value of Planck's constant from the slope of the fit.

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Turn in the rest of this homework as normal via hardcopy in class. You may use calculators/MATLAB to assist you if you wish, but the rest of the work on this assignment will be graded normally.

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5. A photoelectric experiment with Cesium yields stopping potentials for $\lambda = 435.8$ nm and $\lambda = 546.1$ nm to be 0.95 V and 0.38 V, respectively. Using these data only, find the threshold frequency and work function for Cesium and the value of h.

6. One of the funky things that didn't follow the classical expectation for the photoelectric effect is that there was no time-lag between turning on the light-source and measuring a current. In practice – if we don't know about or believe the quantum hypothesis – then we would expect there to be some finite amount of time between when you turn on the source and when a typical electron could gain enough energy from the light beam to be liberated. Let's try and ballpark this expected time-lag for a weak source.

   a) Assume a lightbulb emits total power P equally in all directions. Let us put a metal surface X meters away from this light source, and let's assume it takes energy ϕ to liberate an electron from an atom in this metal. Assuming an atom has a circular cross-section of D, how long would it take for the atom to gain energy ϕ from the source? Leave your answer in terms of P, X, ϕ, and D. (Hint: To check your answer, make sure that it makes sense. The larger P is, the less time it should take. The larger X is, the longer it should take. The higher ϕ is, the longer it should take, and the larger D is, the shorter it should take. That should tell you something about the form of the answer.)

   b) Find the actual value of the time-lag as designed in part a if P is 2 W, X is 0.1 m, ϕ is 6 eV, and D is 0.1 nm.

   c) Assuming that the photon hypothesis is correct (and that photons travel at c), how long would you expect the time-lag to be? Again, assume that $\phi = 6$ eV, that the wavelength of the light is 100 nm, and assume that the path the electron takes through the tube is a straight-line path (no acceleration once liberated). You may ignore relativistic effects for the electron.

7. The Rayleigh-Jeans result for the blackbody energy density can be written as:

$$\rho_T(\lambda)\mathrm{d}\lambda = \frac{8\pi k}{\lambda^5}\lambda T \mathrm{d}\lambda$$

   After introducing the notion of quantized energy, the "correct" (Planck) solution is

$$\rho_T(\lambda)\mathrm{d}\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{\exp(hc/\lambda kT) - 1}\right)\mathrm{d}\lambda$$

   Take the limit of the Planck solution as $T \to \infty$ and demonstrate whether or note these formally agree in the hypothetical limit of infinite temperature. NOTE – this DOES NOT MEAN SHOW THAT YOU GET ∞ for both when $T \to \infty$! This means that we want to show that the Planck solution has the same *functional form* as the Rayleigh-Jeans solution as $T \to \infty$. (Hint – if T is very large, then $hc/\lambda kT$ is small – and it can be treated as a small parameter. Also note that e^x can be written as an infinite series and, when x is small, each successive term of the series is smaller than the previous term).

8. I have a ping-pong ball in my office. It has a diameter of 40 mm (regulation size because I'm a hard-core table tennis player). Let's pretend that I painted it black and it is, for our purposes, a perfect blackbody.

   a) Assume my office is kept at a constant temperature of 293K. How much energy does the ball emit in a year? (You may assume the ball stays in thermal equilibrium at all times).

   b) What would the radius of another blackbody kept at liquid nitrogen temperatures (77K) need to be in order to emit the same amount of power as the ball?

   c) What is the peak wavelength of the blackbody emission spectrum from the ping pong ball?

9. Quite possibly the most famous problem when starting to study blackbody radiation is to estimate the blackbody temperature of the Earth. Let's walk you through this calculation.

   a) Assume the sun is a perfect blackbody at a temperature of, say, 5800K. What is the total energy outputted by the sun per unit time? (Hint – the Stefan-Boltzmann law will give you the amount of power radiated by the sun per unit area. You may also have to look up the radius of the sun).

   b) Assuming a spherical earth of radius R_E a distance R_{SE} from the sun, what fraction of solar radiation hits the Earth's surface? (Hint – what does the Earth look like if you're standing on the sun? Does it look 2 dimensional or 3? What shape would the Earth look like?) For this part of the problem, I am looking for a symbolic – not numeric – answer. Leave your answer in terms of R_E, R_{SE}, and/or fundamental constants.

   c) Based on your answer to the above two questions, how much total power is hitting the Earth's surface at all times? You may have to look up values for R_E and R_{SE}. (I want a numerical answer here).

   d) If we now treat the Earth as a perfect blackbody in equilibrium, this means that the Earth has to radiate away exactly the amount of power you calculated in the above step. (Power in equals power out for an object in thermal equilibrium). Note that, however, the Earth radiates away energy over its whole surface (not just the surface facing the sun). Using the Stefan-Boltmann law, calculate the equilibrium blackbody temperature of the Earth. (Again, I want a numerical answer).

   e) In reality, we've ignored the fact that the Earth is not a perfect blackbody. If we set the albedo of the Earth (the fraction of incoming solar radiation not absorbed by the earth) to be a more realistic value of 0.35 (instead of 0 for a perfect blackbody), what would the equilibrium temperature of the Earth be then? (The power "coming in" is now $(1-A)P_{\text{hitting earth}}$(You may assume the Earth still emits energy like a normal blackbody; it just doesn't absorb it all). (I want a numerical answer here, too.)

   f) Your answer to part (d) was actually a lot closer to the actual average temperature of Earth's surface than your answer to part (e), despite the fact that your description of the Earth was much more accurate in part (e). This is because we're ignoring one more important thing that causes the Earth to warm. What's the big missing part of the picture so far? (Hint – what does Dr. Larsen research?)

10. The maximum blackbody emission occurs when the slope of $\rho_T(\lambda) = 0$. It turns out that we can't actually solve this from first principles because we get something called a transcendental equation. This problem is meant to walk you through how to deal with a situation like this.

   a) First, take the Planck expression for $\rho_T(\lambda)$ and differentiate with respect to λ. You'll have to recall the chain rule and some other stuff. (Be careful with your algebra).

   b) After setting this derivative equal to zero and doing some simplification, you should come up with an expression similar to the following:

$$0 = \text{stuff}\,(\text{different stuff} - n)$$

   with n a prime number. If you divide both sides by "stuff" and make the substitution $x = \frac{hc}{\lambda kT}$, you should be able to rewrite the above expression as:

$$n = \frac{x\exp(x)}{\exp(x) - 1}$$

   This is what we call a transcendental equation, and our usual mathematical tricks aren't particularly helpful here. Enter Computers to help us. There are a number of ways to solve this, but I'm going to discuss one way in particular you can do it.

   Let us rewrite the above expression as follows:

$$f(x) = \frac{x\exp(x)}{\exp(x) - 1} - n$$

   (remember, from part (a) you know what n is supposed to be). One way of finding x is to plot $f(x)$ and find the intersection with the x axis (e.g. the x intercept). Plot $y = f(x)$ in MATLAB and keep zooming in until you can identify where $y = 0$; the x-axis value at that point is what we are looking for. (As a hint, it should be near, but slightly less than, n). For this part of this problem, I do *not* need you to show me your graph or your code; just report the value of x you found from this method.

   c) Using your answer to part (b), note that $x = \frac{hc}{\lambda kT}$. Normally, Wien's law is written in the form $\lambda T =$ constant. Using the value of x you found, find the constant. (It should be very close to the accepted value). (Remember to include units).

11. Star 1 is a perfectly spherical blackbody emitter, with temperature T, radius R, and has a spherical planet of radius r distance D away from the star. Star 2 is a different perfectly spherical blackbody emitter (in a completely different part of the sky) with temperature $3T$, radius $4R$, and has a spherical planet of radius $\frac{r}{2}$ distance $\frac{D}{3}$ away from the star.

    a) What is the ratio $\frac{\text{Total power emitted by star 1}}{\text{Total power emitted by start 2}}$?

    b) What is the ratio $\frac{\text{Power per unit area received by planet 1}}{\text{Power per unit area received by planet 2}}$?

    c) If both planets are also blackbodies, what is the ratio $\frac{\text{Equilibrium temperature of planet 1}}{\text{Equilibrium temperature of planet 2}}$?

    d) If both planets are blackbodies, what is the ratio of $\frac{\text{peak thermal emission wavelength of planet 1}}{\text{peak thermal emission wavelength of planet 2}}$?