Assignment IV, PHYS 230 (Introduction to Modern Physics)
Spring 2017
Due 2/8/17 at start of class

1. A rocket ship is moving directly away from Earth with a speed (as measured by an Earth observer) of $v = \frac{2c}{3}$. The ship launches a probe forward (away from Earth) moving at a speed (as measured by an observer on the ship) of $v = \frac{c}{5}$. The probe then shoots a projectile back towards Earth that moves at a speed (relative to the probe) of $v = \frac{9c}{10}$.
   (a) How fast is the probe moving according to an Earth observer?
   (b) How fast is the projectile moving according to an Earth observer? (and – according to the Earth observer – is the projectile moving towards or away from Earth?)

2. A thin rod of proper length $4a$ is traveling along the x-axis of a frame S with a speed $\sqrt{3}c/2$ in the positive x-direction. A hollow cylinder CD of proper length $2a$ is placed with its axis along the x-axis, so that when the ends of the cylinder are open the rod will pass through the cylinder. The end C of the cylinder is located at $x = -2a$ and the end D at $x = 0$, and both ends are equipped with hypothetical devices capable of closing them off with impenetrable and immovable walls.
   (a) Show that in the frame S in which the cylinder is at rest, both the rod and the cylinder have length $2a$.
   (b) Show that in the frame S' in which the rod is at rest, the length of the rod is $4a$, whereas the length of the cylinder is only a.
   (c) Suppose at time $t = 0$ in frame S the front end B of the rod is located at $x = 0$, and both ends of the cylinder are suddenly closed. Is the rod trapped in the cylinder as might be inferred from the answer to part (a), or is the rod cut in two parts as might be inferred from the answer to part (b)? Reconcile this paradox.

3. How fast must you be moving toward a red light ($\lambda = 650$ nm) for it to appear:
   (a) yellow ($\lambda = 590$ nm)?
   (b) green ($\lambda = 525$ nm)?
   (c) blue ($\lambda = 460$ nm)?

   Leave your answers as a decimal multiple of c (e.g. $0.7241c$).

4. Show that the mass of a particle m is related to its kinetic energy K and momentum p according to the relation:

$$m = \frac{(pc)^2 - K^2}{2Kc^2}$$

5. Show that, for an extremely relativistic particle (in other words, v approaching c), the particle speed v differs from the speed of light c by:

$$c - v \approx \frac{c}{2}\left(\frac{mc^2}{E}\right)^2$$

in which E is the total energy.

6. An electron with rest energy $mc^2 = 0.511$ MeV moves with respect to the laboratory at speed $u = 0.95c$.
   (a) What is γ between the electron and lab frames?
   (b) What is p of the electron (in units of MeV/c)?
   (c) What is the total energy E of the electron?
   (d) What fraction of the electron's total energy is its Kinetic Energy? (It is ok to leave your answer as a decimal; "fraction" here just corresponds to what "portion").

7. (a) Compute the rest energy of a paperclip (estimate or measure the mass somehow).
   (b) If you convert this energy entirely to electrical energy and sell it to your friends and neighbors 13.64 cents per kiloWatt-hour (approximately the average residential price in Charleston according to the National Renewable Energy Laboratory), how much money would you get? (Assume you paid a penny for the paperclip and you found a way to convert the rest energy to electrical energy without any cost to you).
   (c) If you could power a 60 Watt lightbulb with the energy from the paperclip, how long would the bulb stay lit?

8. If the total energy of a particle of mass m is equal to twice its rest energy, then what is the magnitude of the particle's relativistic momentum?

9. A particle moves with speed v at an angle θ with the x-axis in the S frame. Verify that its speed and direction in the S' frame are as given below (assume that the S' frame is moving with velocity u with respect to the S frame):

$$v' = \frac{\left\{v^2 - 2uv\cos\theta + u^2 - \left[\frac{uv\sin\theta}{c}\right]^2\right\}^{1/2}}{1 - \frac{uv\cos\theta}{c^2}}$$

$$\tan\theta' = \frac{\left(\sqrt{1 - \frac{u^2}{c^2}}\right) v\sin\theta}{v\cos\theta - u}$$

10. A particle of mass M decays from rest into two particles. One particle has mass m and the other particle is massless. In terms of M, m, and c, what is the momentum of the massless particle?