Assignment IV, PHYS 272 (MAP)
Fall 2014
Due 9/19/14 at start of class

Note! Although you've already been tested on some of this material, you are still expected to master it and this material may appear on the cumulative final exam for the course or on subsequent tests. Therefore, it is still worth your while to work on these things.

1. It ends up being true that every $n \times n$ matrix can be written as a sum of a symmetric matrix and an anti-symmetric matrix.

   a) Let A be given by:

$$A = \begin{pmatrix} 2 & -3 & 5 \\ 1 & 0 & -3 \\ 2 & 4 & 5 \end{pmatrix}$$

   and find the two matrices (one symmetric, one anti-symmetric) that you must add to get this matrix.

   b) Do the same thing as part (a) for the following matrix B:

$$B = \begin{pmatrix} i & 3+i & 3 \\ \pi & 3 & 1 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

   c) Describe a general algorithm/procedure to divide any matrix M into a symmetric part M_{sym} and an antisymmetric part M_{asym}.

2. Let M and N be given as below:

$$M = \begin{pmatrix} -2 & 3 & 5 \\ 3 & 4 & 6 \\ 4 & -2 & -3 \end{pmatrix} \qquad N = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{pmatrix}$$

   a) Find the trace of M and N.

   b) Find the determinants of M and N.

3. Find the eigenvalues and normalized eigenvectors of the following matrices.

$$R = \begin{pmatrix} -1 & 2 & -2 \\ 2 & 0 & 0 \\ -2 & 0 & -2 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & 2 \\ 0 & 2 & 3 \end{pmatrix} \qquad T = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix}$$

(More on back!)

4. Given the matrix:

$$V = \begin{pmatrix} 1 & 0 & 5i \\ -2i & 2 & 0 \\ 1 & 1+i & 0 \end{pmatrix}$$

Find:

a) V^T

b) V^{-1}

c) V^*

d) $V^\dagger$

5. For each set of functions, use the Wronskian to determine whether the functions are linearly independent.

a) $f_1(x) = x$; $f_2(x) = x^2$

b) $f_1(x) = \sin x$; $f_2(x) = \cos x$

c) $f_1(x) = \sin x$; $f_2(x) = \sin(2x)$

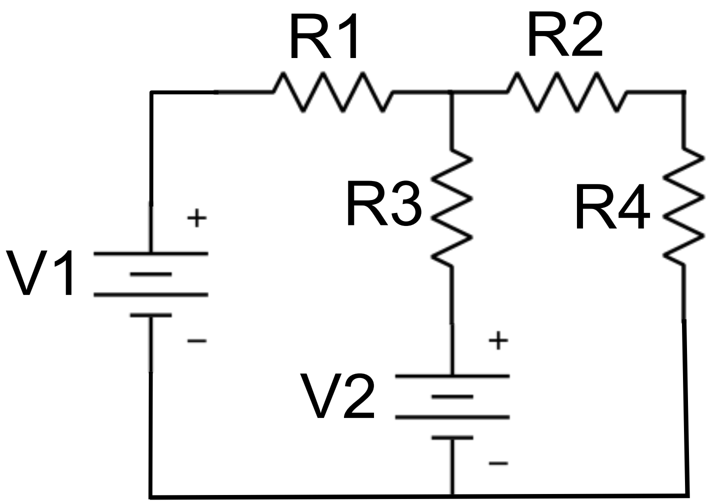
d) $f_1(x) = e^x$, $f_2(x) = \cos x$

e) $f_1(x) = e^{2x}$, $f_2(x) = e^{2(x+3)}$

f) $f_1(x) = 3x^2 + 4x + 2$; $f_2(x) = 5x$; $f_3(x) = 8x + 3$

g) $f_1(x) = \frac{1}{x}$; $f_2(x) = \frac{1}{x^2}$

6. Let's do this problem again, this time without the typo. (If you already have it from the last assignment, then don't worry about it...just attach your previous answer to this homework set and renumber it).



a) Find the current through $R3$. (Hint – there's a reason we're doing this right after we talked about row reduction. You may need to get a refresher on the use of Kirchhoff's Circuit Laws) Hint; you don't necessarily need to know about the current everywhere to get the current through $R3$. Even just finding the one current, it gets kinda messy.

b) Let $V_1 = V_2 = V$, and let $R_1 = R_2 = R_3 = R_4 = R$. Simplify your answer in part (a) to show that, in this case, the current through R_3 is given by $|I| = \frac{V}{5R}$.

More on Back!!!

7. (EXTRA CREDIT! (not required!!)) This is a computationally based extra-credit problem for those of you so inclined. Let's take a matrix M. We know (from class), that $\sum \lambda_i = \text{tr}(M)$, but this is only a tiny, tiny part of the complicated story regarding how eigenvalues depend on the matrix elements. This problem is designed to have you explore the sensitivity of a matrix's eigenvalues based on small perturbations on the matrix elements. We're going to explore it computationally here, but if you want to delve into the theoretical underpinnings, you may want to google "Condition Number".

   Let M be a random, symmetric, positive-definite $3x3$ matrix. To generate one of these in MATLAB, you can use the following code:

   ```
   M=rand(3,3);
   M=5*M*M';
   ```

   The eigenvalues of this matrix can be simply computed by typing the following:

   ```
   eig(M)
   ```

   We want to explore how the eigenvalues change as we perturb one of the elements of matrix M slightly. In particular, here's the question I want you to explore – which matrix element $M_{i,j}$, when perturbed from its initial value, are the resulting eigenvalues most sensitive to a slight change in? (What do I mean by a slight change? let us say that $M_{i,j} = \alpha$, what happens to the eigenvalues if $M_{i,j} = \alpha + \epsilon$ with ϵ a "small" number. How about if $M_{i,j} = \alpha(1+\epsilon)$?) Now, there's a lot to interpret in this question...for example, when I say a "slight change", do I mean a fixed percentage of the actual matrix value, or do I mean an absolute change of less than 0.1, for example? Good question. I'll let you decide how to interpret this. Also – do I mean to ask what is the most critical element for the particular matrix you generated, or do I mean for an "average" matrix (whatever that means)? Good question. I don't know. Finally, what do I mean by "more sensitive"? Do I mean what causes the largest average change in eigenvalue? Do I mean what causes the largest absolute (or fractional) change in one eigenvalue? Do I mean what causes $\lambda_{\text{max}}/\lambda_{\text{min}}$ to change the most? Maybe.

   This is a chance to do some original "research" or "exploration" a-la grad school. I'm not going to give you the exact question. I'm just telling you to play with these basic ideas and report back what you find out. Be clear about what you did, how you did it, and what you think you learned. I'm happy to give you help here, but I'm not really looking for anything in particular – just want to see how you run with a loosely specified problem. The goal isn't to learn anything in particular – it is just to learn *something* about how eigenvalues behave.