Assignment IV, PHYS 301 (Classical Mechanics)
Spring 2014
Due 1/31/14 at start of class

1. Find the center of mass of a rod of length a having a linear mass density:

$$\rho(x) = \begin{cases} 0 & x < 0 \\ k\sqrt[3]{x} & 0 < x < a \\ 0 & x > a \end{cases}$$

(Assume k is a positive constant).

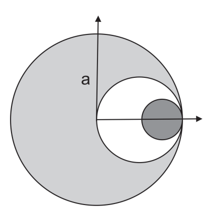
2. Find the center of mass of a hemispherical shell that has a volume mass density:

$$\rho(r) = \begin{cases} 0 & 0 < r < a \\ \frac{c}{r} & a < r < b \\ 0 & r > b \end{cases}$$

(c is a positive constant). For uniformity in writing our answers, assume that the center of the hemisphere is at the origin, and that the hemisphere lies in the region where $z > 0$ (or, if you detest using cartesian coordinates for a system that really should be described with spherical coordinates, let the angle of declination from the "z" axis be constrained so that $\theta \leq \pi/2$).

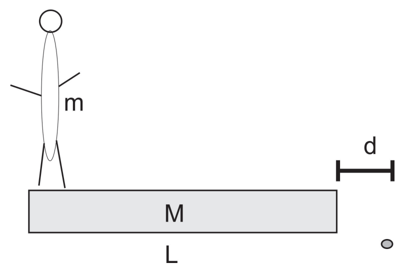
3. A thin circular lamina of radius a has a smaller circle of radius $a/2$ cut from it in the manner shown (so that the edge of the smaller circle is tangent to both the center of the lamina and the edge of the lamina). Inside the cut region, another circular lamina of radius $a/4$ is placed (so that the edge of the smaller lamina is tangent to both the center of the initially removed region and the edge of the hole as shown). If the initial (lighter grey) lamina has uniform mass density ρ_1, the hole has (obviously) mass density 0, and the small (darker grey) lamina has uniform mass density ρ_2:

   a) Find the ratio $\frac{\rho_2}{\rho_1}$ if the center of mass is at the origin of the coordinate system shown.
   b) Find the center of mass of the system in general (leaving ρ_2 and ρ_1 as parameters, not necessarily in the ratio calculated in part (a)).



4. At some point in its trajectory, a projectile of mass M breaks into three fragments. One fragment (with mass $M/5$) continues on with an initial velocity equal to half of the velocity just before fragmentation. The other two fragments (each having mass $2M/5$) go off at right angles to each other with equal speeds. Find the initial speeds of the latter two fragments in terms of the initial speed of the projectile $v_\circ$.

5. A man is standing on the very edge of a raft of length L. The raft has a uniform density and has total mass M. The man has mass m and can safely reach a distance $2d$ beyond the edge of the raft before falling off into the shark infested waters below. A distance d beyond the other edge of the raft lies a mysterious floating bottle with provisions inside. What is the maximum mass m that the man can have and still retrieve the bottle of provisions? Assume the interface between the raft and the surface of the water is frictionless.



6. A man of mass M at rest launches a projectile of mass m horizontally on a frictionless surface. As a consequence, of course, the man recoils.

   a) According to an observer watching from a distance, what fraction of the total kinetic energy T generated by this process is associated with the projectile? (Your answer should be in terms of M and m *only*!)

   b) If you wanted a larger fraction of the energy to go into the projectile and could either add some mass δm to the man or subtract the same mass δm from the projectile, which would result in a larger increase in the fraction of energy in the projectile? (Demonstrate your answer with a clear calculation!)

7. A bug, of mass m, rides on the outer edge of a turntable of radius R. This turntable is in the shape of a disk with constant density and has total mass (without the bug) of M. The bug and turntable spin at an initial angular velocity (about the center of the turntable) $\omega_\circ$. At time $t = 0$ the bug begins to walk in a straight line (from the bug's point of view) towards the center of the turntable with velocity $v_{\rm bug}(t) = \frac{v_\circ r(t)}{R}$. Where $r(t)$ is the bug's current radial position with respect to the center of the turntable. (e.g. $r(t = 0) = R$).

   a) Write an equation for the angular velocity of the turntable as a function of time. You may assume that there are no net torques on the bug/turntable system.

   b) Verify that the angular velocity for $t = 0$ is $\omega_\circ$ and find the angular velocity for $t \to \infty$.

8. Let a projectile of mass m be launched on Earth and have the equation of motion $\vec{r}(t) = (v_\circ t\cos\theta)\,\hat{x} + 0\hat{y} + \left(v_\circ t\sin\theta - \frac{1}{2}gt^2\right)\hat{z}$. (For this problem, we're neglecting air resistance).

   a) Using the implied coordinate system, calculate $\vec{L}(t)$ for the particle.

   b) Take the time derivative of the above to find the torque on the particle. (Remember torque is a vector!)

   c) Find the torque on the particle by calculating $\vec{r} \times \vec{f}$. (Hint; it should match your answer to part (b)!)

9. Two astronauts, each of mass M, play catch with a football in space. The football has mass m and each astronaut is capable of throwing the football with relative velocity v_{rel} *with respect to themselves*. Both astronauts start at rest, and initially astronaut A (on left in the figure as shown) is holding the football. You may neglect any gravitational attraction between the two astronauts.

   a) Let "to the right" be indicated with a positive velocity. After astronaut "B" catches the football for the first time, what are the velocities of astronaut "A" and astronaut "B"? (As always, leave your answer in terms of the variables given in the problem statement only). Signs on this problem (all parts) are likely to get tricky, so leave your answer in terms of just positive variables and indicate if the final velocities are "to the left" or "to the right".

   b) After catching the football thrown by astronaut "A", "B" then throws the football back to astronaut "A". After astronaut "A" catches the football, what are the velocities of astronaut "A" and astronaut "B"? (Just for clarity in the next part of the problem, this is the status after 2 throws).

   c) What are the velocities of the astronauts after N throws? (This took me a surprisingly LONG time; eventually you hope to see a pattern, but it took me longer than I thought it would/should. Don't stress too much about part C if you can't get it; come back to it later after you finish the assignment). I left my answer in terms of v_{thrower} and v_{catcher} since they keep switching roles. (i.e. if we're talking the 4th throw, v_{thrower} corresponds to the most recent thrower, which would be astronaut B).