Assignment IV, PHYS 301 (Classical Mechanics)
Spring 2015
Due 2/6/15 at start of class

1. In lecture (and in your text), it is shown that the distance traveled by a projectile dropped from rest and exposed to both gravity and a quadratic drag follows the formula:

$$y(t) = \frac{m}{c} \ln \left[\cosh \left(\frac{gt}{v_t} \right) \right]$$

Find an approximate expression for $y(t)$ when $t \gg v_t/g$; leave your answer in terms of v_t, m, c, t, and constants. Manipulate your answer in such a way so that g does not explicitly appear (though it can be buried within v_t. (Your final answer should make sense).

2. We just finished studying air resistance in some detail. Now we're going to use our knowledge of momentum conservation to approach the same basic ideas from a different perspective. Assume you have a long, flat sheet of mass M moving at speed V through a region of space that contains small point-particles of mass m moving at speed v. There are c of these point-particles per unit volume in the space. The sheet is moving in a direction parallel to its surface normal. Assume $M \gg m$ and assume that the point particles do not interact with each other in any way. [Note/hint: in perfectly elastic collisions, the relative velocity between two objects before the collision has to equal the magnitude of the relative velocity between the two objects after the collisions].

   a) If $v \ll V$, what is the "drag force" induced per unit area on the sheet due to the point particles?

   b) If $v \gg V$, what is the "drag force" induced per unit area on the sheet due to the point particles? In this case, assume that the component of each particle's velocity in the direction of the sheet's motion is exactly $\pm v/2$. (In reality, these velocities follow a probability distribution function that you'll learn about if you take Thermodynamics. The average speed in each direction, however, is $|\vec{v}|_x = v/2$.)

3. A rod of length L has the following linear mass density:

$$\lambda(x) = \begin{cases} 0 & x < 0 \\ \lambda_\circ + kx^3 & 0 \le x \le L \\ 0 & x > L \end{cases}$$

(Assume $\lambda_\circ$ and k are positive constants).

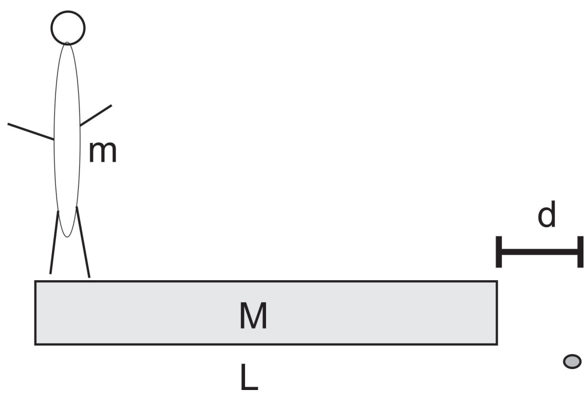
   a) What is the mass of the rod?

   b) Where is the center of mass of the rod?

4. A man is standing on the very edge of a raft. The raft has a uniform density and has total mass M. The man has mass m and can safely reach a distance past the end of the raft of αd ($\alpha > 1$) past his own center of mass before falling off into the shark-infested waters below. A distance d beyond the other edge of the raft lies a mysterious floating bottle with provisions inside.

   a) What is the maximum mass m that the man can have and still retrieve the bottle of provisions? (Assume the raft/water interface is frictionless).

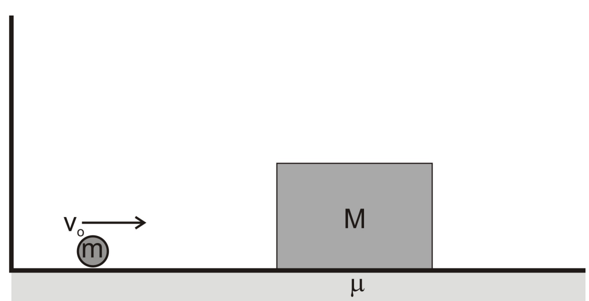
   b) Assuming the man has mass m you calculated in part (a), where is the center of mass of this system? Use an origin where the man starts at $x = 0$ and the other side of the raft is at $x = L$. Your answer should not have an m or and M in it.



5. A ball of mass m and initial speed (to the right) $v_\circ$ bounces back and forth between a fixed wall and a block of mass M with $M \gg m$. The block is initially at rest. Assume that the ball bounces perfectly elastically (both with the wall and with the block). The coefficient of kinetic friction between the block and the ground is μ, whereas there is *no friction between the ball and the ground*. You may work in the approximation where $M \gg m$, and you may assume that the distance between the wall and the block is large enough so that the block has time to come to rest between collisions with the ball.

   a) Show that the ball's speed after n bounces off the block is approximately $\left(1 - \frac{2m}{M}\right)^n v_\circ$.

   b) How far does the block eventually move?

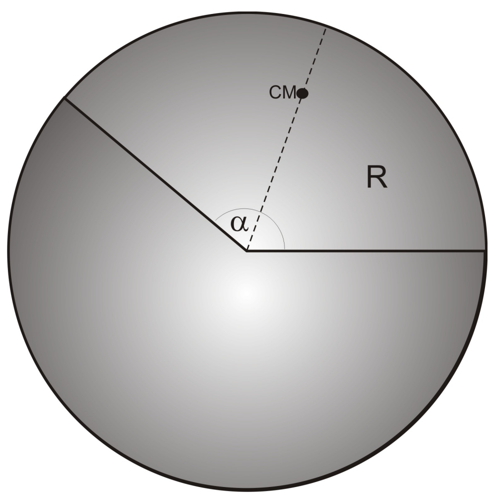


6. A circular sheet of metal is made with that has surface mass density:

$$\sigma = \begin{cases} (ks + \sigma_\circ) & s \leq R \\ 0 & \text{otherwise} \end{cases}$$

with R the radius of the circular sheet, and $\sigma_\circ$ and k some (positive) constants. s is the distance from the origin. From this circular sheet of metal, a wedge is cut out that has central angle α as shown (where $\alpha = \pi$ would indicate that the circle is cut in half, $\alpha = \pi/2$ indicating only a quarter of the circle is retained, etc.; α is constrained between 0 and 2π for obvious reasons). Clearly, the center of mass of the remaining wedge occurs somewhere along the line $\phi = \alpha/2$. Your task is to find how far from the center of the original circle is the center of mass of the wedge cut-out wedge. Leave your answer in terms of k, R, $\sigma_\circ$, and α.

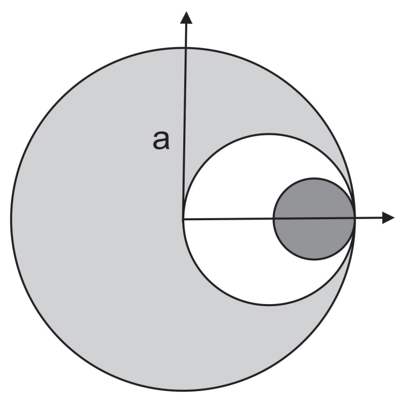
(Hints: If $\alpha = 2\pi$, your answer should be zero, since the center of mass of the whole sheet is at its center. Also, if $\alpha = \pi$ and $k = 0$, your answer should give you $\frac{4R}{3\pi}$ since this is the distance of the center of mass from the center of a uniform semi-circle. Finally, make sure your answer has units of a length!)



7. A thin circular lamina of radius a has a smaller circle of radius $a/2$ cut from it in the manner shown (so that the edge of the smaller circle is tangent to both the center of the lamina and the edge of the lamina). Inside the cut region, another circular lamina of radius $a/4$ is placed (so that the edge of the smaller lamina is tangent to both the center of the initially removed region and the edge of the hole as shown). If the initial (lighter grey) lamina has uniform mass density ρ_1, the hole has (clearly) mass density 0, and the small (darker grey) lamina has uniform mass density ρ_2:

   a) Find the ratio $\frac{\rho_2}{\rho_1}$ if the center of mass is at the left of the darker grey circle.

   b) Find the center of mass of the system in general (leaving ρ_2 and ρ_1 as parameters, not necessarily in the ratio calculated in part (a)).



8. Two astronauts, each of mass M, play catch with a football in space. The football has mass m and each astronaut always throws the football with speed v_{rel} *with respect to themselves*. Both astronauts start at rest, and initially astronaut A (on left in the figure as shown) is holding the football. You may neglect any gravitational attraction between the two astronauts. Give all answers with respect to an observer (astronaut "C") that is initially at rest with respect to both astronauts and remains in this same inertial reference frame throughout the problem. (i.e. give all answers with respect to the only initially relevant reference frame). Note that $v_{\text{rel}} \ll c$, so we don't have to worry about relativistic effects at all.

   a) Let "to the right" be indicated with a positive velocity. After astronaut "B" catches the football for the first time, what are the velocities of astronaut "A" and astronaut "B"? (As always, leave your answer in terms of the variables given in the problem statement only). Signs on this problem (all parts) are likely to get tricky, so leave your answer in terms of just positive variables and indicate if the final velocities are "to the left" or "to the right".

   b) After catching the football thrown by astronaut "A", "B" then throws the football back to astronaut "A". After astronaut "A" catches the football, what are the velocities of astronaut "A" and astronaut "B"? (Just for clarity in the next part of the problem, this is the status after 2 throws).

c) What are the velocities of the astronauts after N throws? (This took me a surprisingly LONG time; eventually you hope to see a pattern, but it took me longer than I thought it would/should. Don't stress too much about part C if you can't get it; come back to it later after you finish the assignment). I left my answer in terms of v_{thrower} and v_{catcher} since they keep switching roles. (i.e. if we're talking the 4th throw, v_{thrower} corresponds to the most recent thrower, which would be astronaut B).

d) At some point, the astronauts move away from each other at a speed that exceeds v_{rel}. At this point, they can't play catch anymore because a thrown ball will never reach the other astronaut. Figure out how many throws this takes. This is rather tricky, so I'll give you a giant hint; the answer takes the general form:

$$N > \frac{\ln(f(m,M))}{\ln(g(m,M))}$$

where $f(m,M)$ and $g(m,M)$ are functions that you're working to find. If you have the right answer, you should find that for $m = 2.5$ and $M = 80$, the astronauts are able to complete 23 passes, but the 24th pass never catches up to the astronaut floating away.