Assignment V, PHYS 308
Fall 2014
Due 10/03/14 at start of class

NOTE: Just like last homework, please leave your answers in terms of actual numbers (with appropriate units) when possible. Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't read it (or I can't follow your reasoning). Extensive exposition on your thought process or strategy is always appreciated.

1. Air flows through a 0.25 inch diameter tube at a rate of 30 liters per minute. Is the flow turbulent? (The transition between laminar and turbulent flow for pipe flow occurs when the Reynolds number hits about 3000). If necessary, assume the ambient temperature is 20 degrees Celcius and the ambient pressure is 1 atm. [This isn't a completely pointless question. In my lab, we have some optical particle counters used for sampling aerosols. These detectors move approximately 30 liters per minute through an opening that is close to 0.25 inches in diameter].

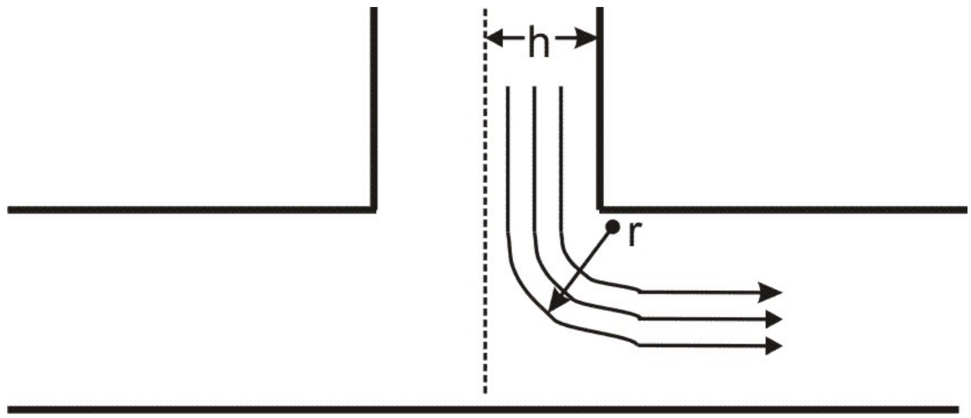
2. In class, I asserted the equation:

$$\tau \frac{\mathrm{d}v_z}{\mathrm{d}t} = \tau g - v_z$$

with initial conditions $v_z(t = 0) = 0$ can be solved via:

$$v_z(t) = \tau g \left(1 - \mathrm{e}^{-t/\tau}\right)$$

   a) Verify that $v_z(t = 0) = 0$ in the above formula.
   b) Verify the proposed solution for $v_z(t)$ satisfies the differential equation.
   c) Find $v_z(t \to \infty)$.
   d) Let's define $v_z(t \to \infty) \equiv v_{\text{term}}$. Develop an expression that gives the time that the particle reaches a speed αv_{term} as a function of α. $(0 \leq \alpha \leq 1)$.
   e) Given your answer to part (d), how long would it take a 100 nm diameter aerosol particle (with density 1300 kg/m^3) to reach 95% of its terminal fallspeed? (Use 1.2 as the Knudsen number for a 100 nm aerosol particle, and assume we can just use modified Stokes' drag (e.g. $F_{\text{drag}} \propto v$).
   f) How far would the aerosol particle have fallen in part (e) in the time it takes to reach 95% of its terminal fallspeed? (Your answer is likely to surprise you. Hint – if your answer to part (e) is T, then the distance fallen after time T can be calculated from $\Delta z = \int_0^T v_z(t)\mathrm{d}t$.)



3. The picture shown above shows a simplified version of an aerosol sampling device known as an "impactor". For the simplified geometry shown above, you may assume that the flow is uniform near the exit and that the streamlines of airflow are arcs of a circle with centers at the marked point. Some particles in the flow will move across streamlines and may be deposited on the surface.

   a) Show that particles of a given diameter, D_p, will move with constant radial velocity $v_r = (\tau U^2)/r$ where r is the radius of curvature of the streamline, τ is the relaxation time, and U is the speed of the gas. (You may assume U is constant).

   b) Show that in traveling along the arc, the particle is displaced by a total radial distance $\Delta = (\pi/2)\tau U$.

   c) Show that the fraction of particles that are deposited is $f = \frac{\pi\tau U}{2h}$. (You may assume that the concentration of aerosols is spatially uniform in the incoming flow).

4. To calibrate a rain-sensing device that we have (a 2-dimensional video disdrometer), we have a process by which we throw spherical steel ball-bearings through the device from a known height. I'm not 100% sure the exact height, but – for the sake of this problem – let's say that it is 60 cm above the sensing surface. Let's say we are dropping 10 mm spheres made out of solid steel (density of 8050 kg/m^3), initially at rest (hence "dropping") from a height of 60 cm through air (near the surface of the Earth). For these spheres, we ignore both wind and drag. Let's make sure this is reasonable.

   a) Assuming $C_D = 0.44$, what is the terminal velocity (in air) of these spheres?

   b) If you neglect air resistance entirely, the downward velocity of a falling object is $v = v_\circ + gt$ and, in particular, if dropped, you have $v = gt$. Use the formula derived in class to find the numerical ratio $v(t)/(gt)$ for these spheres for (i) $t = 0.1$ second, (ii) $t = 0.5$ seconds, (iii) $t = 1.0$ seconds, (iv) $t = 2.0$ seconds, (v) $t = 5.0$ seconds, and (vi) $t = 10$ seconds. (I personally used MATLAB to help me out here. Saved me a bunch of computation. You may solve this however you'd like.)

   c) Based on your answers to (a) and (b) (or other information, if necessary), explain why we don't have to factor in drag for these spheres.

5. (Extra Credit!) Use your favorite computer algebra system/coding language/computational resource to draw a log-log plot of F_{drag} as a function of particle size. Have curves for Stokes' drag (dotted line) and the empirically adjusted value (solid line) as presented in class. You may assume that the fluid is air (near the surface of the Earth), the relative velocity is 10 m/s, and have the x-axis (particle size) range from 0.1 nm to 1 cm. Note that this is 8 orders of magnitude difference in size, so you don't want to have a step size of 0.1 nm unless you have a week of computer time to kill. (Ask in class for hints!) In addition to the graph, please turn in your code/mathematica session/maple desktop/matlab code/excel spreadsheet/etc. (Feel free to assume $C_C = 1$ throughout the entire range of sizes, even though that's a bit of an oversimplification).