Assignment V, PHYS 230 (Modern Physics)

Fall 2019 Due Thursday October 24th, 2019 at Start of Class

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

1. In the handout to the class, Dr. Larsen started with the Bohr postulates and derived the following for the velocity, energy, and radius of the ground state of a single electron atom:

$$v_\circ = \frac{Ze^2}{4\pi\epsilon_\circ\hbar}$$

$$E_\circ = \frac{-m}{2}\left(\frac{Ze^2}{4\pi\epsilon_\circ\hbar}\right)^2$$

$$r_\circ = a_\circ = \frac{4\pi\epsilon_\circ\hbar^2}{Ze^2 m_e}$$

Note that this notation is a little weird, because $v_\circ$ is the velocity for the $n = 1$ state, so $v_\circ$ and v_n with $n = 1$ would give you the same thing. Use the same basic procedure that Dr. Larsen used for the ground state to find expressions for v_n, E_n, and r_n for any state of a single electron atom. (As a simple check, your expressions should reduce to the expressions for $v_\circ$, $E_\circ$, and $r_\circ$ when $n = 1$).

2. For a Hydrogen atom in the Bohr model:

a) What would the velocity of a ground state electron be (I'm looking for a numerical answer here)?

b) What would the velocity of an electron in the $n = 12$ excited state be?

c) Since the velocity of an electron is clearly highest in the ground state (if you didn't get that, your answer to the previous problem is likely incorrect), what is the value of γ (from relativity) for a ground state electron in Hydrogen? Use your computed value to comment on the need to introduce relativistic corrections into Bohr's model.

d) A hydrogen atom exists in an excited state for typically around 10 nanoseconds. How many revolutions would Bohr's theory suggest that an electron would make in an $n = 3$ state before returning to the ground state?

3. In the Lithium atom ($Z = 3$), two electrons are in the $n = 1$ orbit and the third is in the $n = 2$ orbit. (Only two are allowed in the $n = 1$ orbit because of the exclusion principle, which will be discussed in chapter 7). The interaction of the inner electrons with the outer one can be approximated by writing the energy of the outer electron as:

$$E = -(Z')^2 \frac{E_1}{n^2}$$

where $E_1 = 13.6$ eV, $n = 2$, and Z' is the effective nuclear charge, which is less than 3 because of the "screening" effect of the two inner electrons. (Since there are negative charges between the outer electron and the nucleus at least part of the time, the outer electron doesn't feel the total net effect of the full nuclear charge). Using the measured ionization energy of 5.39 eV needed to remove the outermost electron, calculate Z' for this atom.

4. The formulation of much of the wave mechanics that will appear in the coming weeks relies on the formulation of a "wave equation". The most familiar wave equation is a differential equation of the form:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

where v is the velocity of the wave, and $u(x,t)$ is a function describing the thing that's waving. Although many people in this class have not had differential equations – so "solving" this isn't necessarily fair for me to ask....there *are* things we can do with this equation that might help us understanding what's going on a bit better.

a) Verify (by explicitly taking the necessary derivatives) that $u = A\cos(kx - \omega t)$ is a solution to this equation. (In doing this, you will find a relationship that must be obeyed between k, ω, and v for this to be a valid solution. What is that relationship?) Treat A, k, and ω as real constants.

b) Verify (by explicitly taking the necessary derivatives) that $u = Be^{i(kx-\omega t)}$ is a solution to this equation as well. (Again, you'll find the same relationship between k, ω, and v.) Treat B, k, and ω as real constants.

5. In the coming lectures, we're going to introduce the notion of a "wave packet". This problem is designed to let us start dealing with the mathematics related to such a wave packet.

   Recall the sum/difference formulas for sines and cosines:

$$
\begin{aligned}
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\
\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta
\end{aligned}
$$

   (Although I gave these to you, these are some of the very few trig identities you should have memorized; using these and $\sin^2 x + \cos^2 x = 1$, you can derive pretty much all of the other trigonometric identities).

   a) Consider $\cos(\alpha x \pm \delta x)$ where $\delta/\alpha \ll 1$. Using the above formulas, what do you obtain if you add $\cos(\alpha x + \delta x)$ to $\cos(\alpha x - \delta x)$?

   b) In your answer to part (a), you should have come up with an expression that is the product of two trig functions. Use MATLAB to make a plot of the resulting function using $\alpha = 10$ 1/m and $\delta = 0.50$ 1/m for $0 \leq x \leq 40$ m; make the line black with a line-width of 3 points. On the same axes, plot $2\cos(\alpha x)$ in red (thinner line width) and $2\cos(\delta x)$ in blue (also thinner line width). (No need for a y-axis label or legend for this figure). Just like the last homework, send me the .m file that generates the figure to LarsenML@gmail.com

   c) When looking at the graph generated in part (b) above, you should see that there is a clear dual periodicity to the resulting wave – there are two different wavelengths that are identifiable in the same curve. In terms of α and δ, what are these two wavelengths? Indicate which one is the longer of the two wavelengths by putting an asterisk next to it.