PHYS 230 - Assignment 5

Spring 2017
Due Wednesday, February 22nd, 2017

This assignment is almost purely computational in nature. It is expected that you will utilize Mathematica to complete the work on this assignment. As such, we're going to use a different paradigm to turn in your work than we normally do. Rather than turn in your work on paper, please email me your workbook (which should be named something.nb) prior to the beginning of class on February 22nd. My email address is LarsenML@cofc.edu Please make sure you have your name on the document somewhere.

Extra Details

Please make sure that your code actually works when you pull it up in a fresh Mathematica session; that's how I will be grading it. (Even if it works in your current session, that's no guarantee that it will necessarily work when executed in order by me.)

Please also try to keep your workbook clean and, when appropriate, please include text. I'm writing both the assignment and the answer key in Mathematica, and I never use this tool. If I can do it (and put in extra text), you can as well.

Note that beneath each part of each problem, you will see "Answer:" Don't let that throw you. I made this homework in Mathematica, and when I distribute the answer key, I will do so electronically in a way so that you can just open up each section of the Mathematica notebook and see my answer. For now, just do what the problem asks you to.

Problem 1

As you (hopefully) remember, the expression for gamma (used extensively in special relativity) can be written as $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$. In this problem, we will be using this function as a basic template to do some relatively straightforward things.

Part A

Make a plot of γ as a function of v. Make sure to label your axes, and make your displayed y values go from 0 to 5.

Answer:

Part B

Make a list that gives gamma for the following values of v: 0.03c, 0.10c, 0.50c, 0.75c, 0.95c, 0.999c, and 0.99999c.

Answer:

Part C

Find v such that γ(v)=42. I want the answer to be the speed (in meters per second) and I want it accurate to a total of 10 digits precision. (If you can, see if you can figure out how to suppress the negative result, since speeds are always positive).

Answer:

Problem 2

We will soon be talking about some other, new content. Although this may not be something you know much about yet, we can use Mathematica to play with some of the related functions.

We will be (briefly) looking into a Phenomena known as Blackbody Radiation. It has been shown that the energy density inside something called a blackbody is related to the Kelvin temperature of the object. The so-called blackbody curve gives us an expression for the total energy density inside a blackbody as a function of wavelength for a particular temperature. The mathematical expression for this is:

$$\rho(\lambda) = \frac{8\pi h c}{\lambda^5}\left(\frac{1}{\exp[\frac{hc}{\lambda k T}]-1}\right)$$

where ρ is the energy density function, T is the Kelvin temperature of interest, exp[ ] is a standard notation that merely means “e to the thing in brackets”, h is a numerical constant called Planck’s constant (which has a value of 6.626 x 10^{-34} Js -- the units being Joules times seconds) and k is a numerical constant known as Boltzmann’s Constant which has a numerical value of 1.38 x 10^{-23}J/K (Joules per Kelvin). c is, of course, the speed of light.

For certain wavelengths, this relationship can be approximated using the Rayleigh-Jeans relationship:

$$\rho(\lambda) = \left(\frac{8\pi k}{\lambda^5}\right)\lambda T$$

Part A

Plot both the Planck Energy density formula and the Rayleigh-Jeans relationship on the same plot for T=3000K. (You probably want to use a “LogLinearPlot”). Use a range for the wavelength (λ) from 200 nm to 10000 nm. You may have to manually set the limits for the y-axis to actually see the interesting parts of both curves. (Experiment a little if you have to).

Answer:

Part B

Now make a manipulate-able plot of just the Planck Energy density spectrum (on a "LogLinearPlot") where the user can change and/or animate T over a range from about 77K (the temperature of liquid nitrogen) to 6000K (about the temperature of our sun). IF you can, try to make it so that the y-axis stays fixed as the animation proceeds. (If you don't do this, you will have the y-axis constantly changing as you manipulate the temperature so it becomes harder to see what's going on). Use a wavelength range from 100 nm to 1 mm.

Answer:

Part C

The answer to the previous part might be hard to see anything interesting for low temperatures if you also want to see the behavior at high temperatures on the same y-axis scaling. As such, try using a LogLogPlot instead. Go ahead and let the y-axis change this time as you animate things, but -- as you run the animation -- pay attention to the values you end up with on the y-axis.

Answer:

Part D

We can integrate the Blackbody curve to determine what fraction of all energy inside the blackbody exists at wavelengths that we can see. If the minimum wavelength we can see is $\lambda 1$ and the largest we can see is $\lambda 2$, we can find this fraction by taking $\frac{\int_{\lambda 1}^{\lambda 2} \rho(\lambda)\, d\lambda}{\int_0^{\infty} \rho(\lambda)\, d\lambda}$. Calculate this ratio (aka fraction) for $\lambda 1$=400 nm and $\lambda 2$=700 nm and T=6000K. (Make sure to convert your wavelengths to SI units!)

Answer:

Problem 3

As long as we're talking about Blackbody radiation, we might as well do some related math we will need. There's a relationship known as Wien's law which tells us the wavelength of peak emission.

Since we have an expression for $\rho(\lambda)$, you might this would be mathematically easy. We merely would take the derivative of ρ with respect to λ, set the answer equal to 0 (to indicate we are at an extremum) and find the value of λ that gives us $\frac{d\rho}{d\lambda}$=0. Turns out, if you try to do that, you get something called a "transcendental equation" that can't really be solved by hand. I'll leave out some of the gory details, but the equation you ultimately have to solve looks something like the following:

$$\frac{(x\,\mathrm{Exp}[x])}{\mathrm{Exp}[x]-1} - 5 = 0$$

where, again, Exp[x] indicates taking e to the power x. In this problem, we're going to try and use Mathematica to figure out what x has to be so that this equation is true. We'll do this a couple of ways.

Part A

One way to do this is sort of a guess and check, but we can do better. Let's define the left hand side of the function above as f(x). Then, we will plot f(x) as a function of x and look for the x-intercept of the plot. This should give us a pretty good estimate of what x must be. You should try to find this value by making a series of plots. The domain of your first plot should go from 0 to 10. Then, keep making new plots with a smaller range of x-value until you have a pretty good idea of what the value of x that solves this equation must be. You will be graded on the series of plots you provide.

Answer:

Part B

We can also do this by trying to use a numerical approximation using the solve function. Use the Solve function to get the answer to at least 10 digits of precision.

Answer: