Assignment V, PHYS 272 (MAP)
Fall 2014
Due 9/26/14 at start of class

1. The following matrix is symmetric, therefore it should have orthogonal eigenvectors.

$$A = \begin{pmatrix} 3 & -1 & 3 \\ -1 & 7 & -1 \\ 3 & -1 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} -3 & 4 & 3 \\ 4 & 5 & 1 \\ 3 & 1 & 12 \end{pmatrix}$$

   a) Find the eigenvalues and the (normalized) eigenvectors for matrix A.
   b) Explicitly show the eigenvectors for matrix A are mutually orthogonal to each other.
   c) Find the eigenvalues and the (normalized) eigenvectors for matrix B.
   d) Explicitly show the eigenvectors for matrix B are mutually orthogonal to each other.

2. We mentioned in class that the following matrix describes a rotation via angle ϕ with respect to the z axis:

$$M = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   Find the eigenvalues and eigenvectors of the above matrix, and comment on them. (Do they make sense? Why or why not?)

MORE ON BACK!

Vector Calculus Review Time!!!

Let:

$$\begin{aligned}
\vec{a} &= yz\hat{x} + xz\hat{y} + xy\hat{z} \\
\vec{b} &= x^2 e^{-z}\hat{x} + y^3 \ln(x)\hat{y} + z\cosh(iy)\hat{z} \\
\vec{c} &= r^2\hat{r} \qquad \text{(spherical coordinates)} \\
\vec{d} &= \frac{1}{s^2}\hat{s} \qquad \text{(cylindrical coordinates)} \\
f &= 3x^2yz \\
g &= \cosh(xy) \\
h &= r^3 \qquad \text{(spherical coordinates)}
\end{aligned}$$

3. Compute the following:

a) $\vec{\nabla} \cdot \vec{a}$

b) $\vec{\nabla} \cdot \vec{b}$

c) $\vec{\nabla} \cdot \vec{c}$

d) $\vec{\nabla} \cdot \vec{d}$

e) $\vec{\nabla} \times \vec{a}$

f) $\vec{\nabla} \times \vec{b}$

g) $\vec{\nabla} \times \vec{c}$

h) $\vec{\nabla} \times \vec{d}$

i) $\vec{\nabla} f$

j) $\vec{\nabla} g$

k) $\vec{\nabla} h$

l) $\nabla^2 f$ (if you haven't seen ∇^2 before, this means $\vec{\nabla} \cdot (\vec{\nabla} f)$).

m) $\nabla^2 g$

n) $\nabla^2 h$