Assignment V, PHYS 301 (Classical Mechanics)
Spring 2014
Due 2/7/14 at start of class

(Note: This is the last homework before your first exam. However, we will continue discussing content that will be fair game to appear on your first exam after this homework is due. If you are unsure of where the testable material stops, please ask Dr. Larsen).

1. The potential energy of two atoms in a molecule can sometimes be approximated by the Morse function:

$$U(r) = A\left[\left(\mathrm{e}^{(R-r)/S} - 1\right)^2 - 1\right]$$

where r is the distance between the two atoms and A, R, and S are positive constants with $S \ll R$.

  a) Find the separation where $U(r)$ is at a minimum.
  b) Define the equilibrium position of the potential function as $r_\circ$. Explore $U(r_\circ + \delta r)$ (with δr a small parameter). In particular, you should be able to write the potential near $r_\circ$ in the form $U(r + \delta r) \approx C + \frac{1}{2}k(\delta r)^2$ with C and k some constants. Find C and k. (Note that this looks like the potential energy from a spring).

2. A particle of mass m moves in one dimension; the force can be described as:

$$F(x) = -4C\left[\frac{x}{a^2} - \frac{x^3}{a^4}\right]$$

(where I neglect the vector sign since this is a one-dimensional system; positive indicates a force towards larger x). If it isn't clear, this is a conservative force.

Find:

  a) The potential energy function associated with this force.
  b) Hand sketch a plot of $U(x)$. (Take care to mark important points, label your axes, mark any asymptotes, etc. We're looking mostly for a quantitative picture here, but if there are any important points on either axis, the values at those points should be clear).
  c) Find any positions of equilibrium; identify each equilibrium point as a stable or unstable equilibrium point.
  d) What is the minimum speed the particle would need at the origin to escape to infinity?

3. A solid triangular wedge is placed on earth. The hypotenuse (length D) has a surface that is frictionless. To the top of the wedge is affixed a massless spring with spring constant k_1 and equilibrium length $x_\circ$. To the bottom of the wedge, a massless spring with spring constant k_2 and equilibrium length $x_\circ$ is placed. A mass m is attached to both springs. (You may assume that the mass has zero size and, as such, if this experiment was conducted in the absence of gravity, neither spring would be extended or compressed and the mass would be at equilibrium at a distance $D/2 = x_\circ$ from either edge of the hypotenuse). The angle of inclination of this wedge is ϕ as shown. Let the variable ℓ indicate the distance of the mass from the bottom right corner of the triangle.

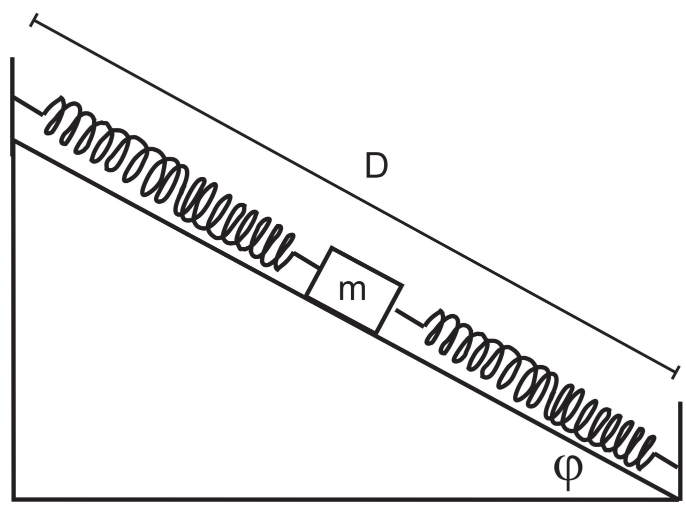
a) Letting $\ell = x_\circ$ correspond to the case $U(\ell) = 0$, write down a potential energy function for this system (i.e. write down $U(\ell)$). (Make sure that you get $U(\ell) = 0$ when $\ell = x_\circ$!)

b) Find the net force on the block as a function of ℓ via $\vec{F} = -\vec{\nabla} U$.

c) Find the equilibrium position of the block (where net force is zero) as a function of $x_\circ$, ϕ, k_1, k_2, m, and g only. (Hint / something to check on your final answer: What should your equilibrium length be when $\phi = 0$?)

d) Show that this equilibrium calculated in part (c) above is a stable equilibrium.

e) If the block is pulled to the position $\ell = \frac{3x_\circ}{2}$ and released, what is the block's *speed* as it moves through the point $\ell = x_\circ$? (Leave your answer in terms of $x_\circ$, ϕ, k_1, k_2, m, and g only.)

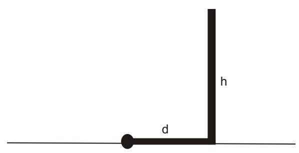


4. Two pipes of lengths d and h are joined at a right angle as shown below. Each pipe has uniform density and negligible cross-sectional area. The pipe of length d has mass m and the pipe of length h has mass M. As shown, the pipe of length d currently lies on level ground.

   a) Find an expression for $U(\phi)$ where ϕ is the angle the pipe of length d makes with the horizontal. (The picture is drawn with $\phi = 0$). Your expression should be a function of m, M, d, h, and ϕ only.

   b) Find ϕ such that the system is in equilibrium. You may leave your answer in terms of inverse trig functions.

   c) If $M = m$ and $d = D$, find ϕ in terms of either an arcsin or arccos.



5. A mass hangs straight down from a massless "super-spring" (patent pending) attached to the ceiling that actually has a restoring force $F = -\alpha(x - x_\circ)^{3/2}$ (a 3/2 dependence instead of a linear dependence for all other springs in the known universe). Alpha is a positive constant. Find:

   a) The equilibrium position of the mass with respect to the ceiling.

   b) Identify if this equilibrium is stable or unstable.

6. Look at the figure below, which is a picture of a "weeble" (a child's toy; they wobble, but they don't fall down). They are solid objects that have a hemispherical base of radius R, with another shape above it. The center of mass is some distance d above the bottom of the weeble. (You can assume d is a known quantity).

   a) Write a potential energy function $U(\theta)$ for the weeble in terms of m, d, R, g, and θ, where θ is the angular rotation of the weeble from the upright position. (You may assume $|\theta| \leq \pi/2$ so that the contact point of the weeble and the surface is still on the hemisphere).

   b) There is only one equilibrium angle for this system. It should be easy to find (if nothing else, just from symmetry). The behavior at the equilibrium angle can be either stable or unstable, however. Find a relationship between d and R that constrains the equilibrium to be a stable one. Interpret this physically.