Assignment V, PHYS 301 (Classical Mechanics)
Spring 2017
Due 3/3/17 at start of class

1. The force acting on a particle of mass m is:

$$\vec{F} = k\left[(2xy + x^2)\hat{x} + (x^2 + y^2)\hat{y} + z^2\hat{z}\right]$$

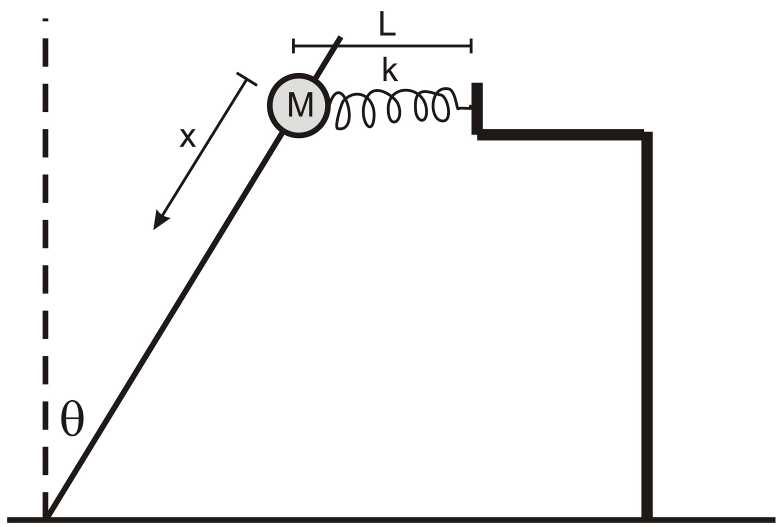
with k a positive constant.

a) Show that $\vec{F}$ is a conservative force.

b) What is the associated potential energy function for the force $\vec{F}$? Assume that the potential energy at the origin is $U_\circ$.

c) What work is done by the force $\vec{F}$ as the particle goes from the points $\hat{x} + 2\hat{y} + 3\hat{z}$ to the point $-3\hat{x} + 4\hat{y} - 5\hat{z}$?

2. A frictionless pole is mounted and tilted at an angle of θ from the vertical. Sliding on this pole is a mass M, which is connected via a spring of spring-constant k to a fixed point. When the mass is at the same height as the spring-mount, the spring is at its equilibrium length L. Let x be the distance measured down the pole, with $x = 0$ defined as the point where the spring is unstretched. (See picture below). Write down a potential energy function $U(x)$ for this system. Leave your answer in terms of x, m, g, L, k, and θ only.



3. A solid triangular wedge is placed on Earth. The hypotenuse (of length D) has a surface that is frictionless. To the top of the wedge is affixed a (massless) spring with spring constant k and equilibrium length $x_\circ$. To the bottom of the wedge, a massless spring with spring constant $3k$ and equilibrium length $x_\circ$ is placed. A point mass m is attached to both springs. $D = 2x_\circ$ so that, if this system were placed in space (in the absence of gravity), the system would be in a stable equilibrium with the point mass exactly halfway down the hypotenuse. The angle of inclination of this wedge is ϕ as shown. Let the variable ℓ indicate the distance of the mass from the bottom right corner of the triangle.

a) Letting $\ell = x_\circ$ correspond to the case where $U(\ell) = 0$, write down a potential energy function for this system (e.g. write down $U(\ell)$. (Make sure that you get $U(\ell) = 0$ when $\ell = x_\circ$!

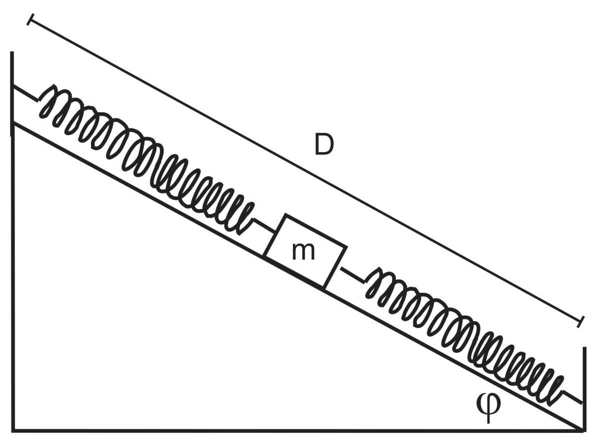
b) Find the net force on the block as a function of ℓ via $\vec{F} = -\vec{\nabla} U$.

c) Find the equilibrium position of the block as a function of $x_\circ$, ϕ, k, m, and g only. (Hint / something for you to check – what should your equilibrium ℓ be when $\phi = 0$?)

d) Show that the equilibrium calculated in part (c) is a stable equilibrium. Justify your answer with computations.

e) If the block is pulled to the position $\ell = \frac{5x_\circ}{3}$ and released, what is the block's speed as it moves through the point $\ell = x_\circ$? (Leave your answer in terms of $x_\circ, \phi, k, m,$ and g only.)

f) If the mass is displaced from its equilibrium position by a small amount, what would the angular frequency of the resulting small oscillations be? (In terms of variables in the problem statement, of course).



4. In class, I argued that the half-width half-maximum distance on a resonance plot was β. Show that A^2 drops to half of its maximum value when $\omega \approx \omega_\circ \pm \beta$. (You may have to make an approximation or two....)

5. A particle of mass m is subject to a spring force $-kx$ and also a driving force $f_\circ m \cos(\omega t)$; however there is no damping force.

   a) After rewriting this system, you end up with $\ddot{x} + \omega_\circ^2 x = f_\circ \cos(\omega t)$. Find a particular solution to this differential equation by guessing $x(t) = C\cos(\omega t) + D\sin(\omega t)$. Your answer should be in terms of $\omega_\circ$, ω, and $f_\circ$ only.

   b) Based on your answer to part (a), you should be able to come up with the general solution to $x(t)$ pretty easily. From this solution, consider the following: If this system had initial conditions $x(t=0) = \frac{3f_\circ}{(\omega_\circ^2 - \omega^2)}$ and $v(t=0) = 0$, what is the acceleration of this system be at $t = 0$?

6. A weakly damped harmonic oscillator ($\beta \ll \omega_\circ$) consists of a mass m stretched a distance $A_\circ$ from equilibrium and released at time $t = 0$. Find and approximate expression for its energy as a function of time. (Only keep terms to first order in the small parameter $\beta/\omega_\circ$, and leave your answer in terms of k, β, $A_\circ$, $\omega_\circ$, and t only.)

7. (This one is stolen shamelessly from your text. I copy it here just in case you don't have a copy of the textbook). When a car drives along a "washboard" road, the regular bumps cause the wheels to oscillate on the springs. (What actually oscillates is each axle assembly, comprising the axle and its two wheels.) Find the speed fo my car at which this oscillation resonates, given the following information:

   a) When four 80-kg men climb into my car, the body sinks by a couple of centimeters. Use this to estimate the spring constant k of each of the four springs.

   b) IF an axle assembly (axle plus two wheels) has total mass 50 kg, what is the natural frequency of the assembly oscillating on its two springs?

   c) If the bumps on a road are 80 cm apart, at about what speed would these oscillations go into resonance? (Convert your answer to miles per hour).