Assignment V, PHYS 308
Fall 2016
Due 9/30/16 at start of class

NOTE: Just like last homework, please leave your answers in terms of actual numbers (with appropriate units) when possible. Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't read it (or I can't follow your reasoning). Extensive exposition on your thought process or strategy is always appreciated.

1. Surface air is at 17 Celcius where the dew point is 5 Celcius. Approximately how high up is the LCL?

2. A stable air parcel can be found at an altitude of 3.5 km. The relative humidity of this parcel at this altitude is still quite small, so there is no chance of water condensing in the parcel. The local environmental lapse rate at this altitude is 2 Kelvin/km, and the current temperature of the ambient air at this altitude is about 255K. The stable air parcel is displaced from its equilibrium height by 50 meters at time $t = 0$.

   a) What is the frequency (not angular frequency!) of oscillation about its equilibrium height for this parcel?

   b) If the air parcel is advecting (moving sideways due to the steady wind) at a velocity of 6 m/s, what is the physical wavelength of the oscillation?

   c) If the environmental lapse rate were 4 Kelvin/km, how would your answers to parts (a) and (b) change? (You don't have to calculate it; just let me know for both quantities whether it would increase, decrease, or stay the same.)

3. A rainstorm passes a level parking-lot and deposits a liquid water layer that is 5 mm thick. Assume all of this water is re-evaporated into the atmosphere (ignore any temperature change and just use the latent heat of vaporization I gave you in class). How long would the sun have to shine directly on the parking-lot in order to absorb the energy required to evaporate this water? [Remember, not all of the sun's radiation is absorbed by the surface of the Earth.] You may assume that the parking-lot is aligned perpendicular to the vector pointing from the Sun to the Earth. Clearly outline your reasoning and assumptions! If you look up any constants, let me know where you got them from.

4. An average raindrop is 1 mm in diameter. An average cloud drop is about 10 μm in diameter. How many cloud drops does it take to make a raindrop?

5. I have (or will) mention a number of times in class that showing the frequency distribution of atmospheric particulates as a function of size (e.g. a "size-distribution") can be deceiving. This is because many atmospheric processes depend on looking at variables besides particle number. It is often more important to consider the total surface area as a function of size, or total volume as a function of size. This problem is designed to start getting you to think about this.

   Let us start by thinking about what happens to the total surface area when you divide a given volume of water among spherical drops of increasingly smaller size.

   a) If you have a single spherical drop of volume $V_\circ$, what is its surface area? (Please simplify your answer, and leave your answer in terms of $V_\circ$ (NOT r)!

   b) If you divide the volume $V_\circ$ into n spherical drops, each of volume $V_\circ/n$, the total surface area of the collection of spherical drops can be written in the form $f(n)\cdot$S.A. where S.A. is the answer to part (a). Find $f(n)$.

   c) The radar reflectivity factor can be written as $Z = \sum_i \kappa D_i^6$ with i summing over all of the drops in question, κ some (for now unspecified) constant and D_i corresponding to the drop radius of the ith drop. If you again divide the liquid volume $V_\circ$ into n spherical drops, each of volume $V_\circ/n$, find an expression for $Z(n)$.

   d) Let us push the previous scenario just a little bit further. Let us say that half of the volume ($V_\circ/2$) is separated into n_1 equal volume drops. The other half of the volume is separated into n_2 equal volume drops. Find an expression for $Z(n_1, n_2)$.

   e) In Physics, we often are concerned with obvious limiting cases. Compare the expression you developed in part (c) above to the expression you developed in part (d) above. To do this, define $N = n = n_1 + n_2$. In that case, if you've done parts (c) and (d) properly, you should see that $Z(N/2, N/2)$ (your answer to part (d) with both n_1 and $n_2 = N/2$) is equal to $Z(N)$ (your answer to part (c) with $n = N$). Verify that this works, and rewrite your answer to part (d) as $Z(n, N - n)$.

   f) Take the derivative of the expression you derived in part (e) above with respect to n. You should find that you hit an extremum when $n = N/2$. Verify this.

   g) Does the extremum of $Z(n, N - n)$ found when $n = N/2$ in the previous problem correspond to a min or max for $Z(n, N - n)$?

   h) Interpret your answer to part (g). What does this mean physically?

6. The maximum legal concentration of $PM_{2.5}$ particles in air is now 12 micrograms per cubic meter of air. ($PM_{2.5}$ particles are particulate matter particles that are at least 2.5 micrometers in diameter). (Note – the maximum value presented above is a yearly average maximum; the instantaneous value is allowed to be higher than this). For simplicity, assume all $PM_{2.5}$ particles are 2.5 micrometers in diameter and also assume they are all made of liquid water. What is the maximum *number* concentration of $PM_{2.5}$ particles in the atmosphere in each cubic centimeter? (i.e. how many particles per cubic centimeter of air is the same as 15 micrograms per meter cubed if all the particles are spherical and have a diameter of 2.5 micrometers and the density of water (1g/cc)?) (The simplifying assumptions we are making are obviously false, but our goal is to get an approximate value here. Since most real aerosols are a little denser than liquid water and since most $PM_{2.5}$ particles are substantially smaller than 2.5 micrometers, our computation in this problem will be something of an underestimate).

7. The terminal fall-velocity of a 10 nanometer aerosol particle is about 5 nm/s. 10 nm aerosol particles can easily be found several kilometers up in the atmosphere, yet their residence time (the amount of time they stay airborne before leaving the atmosphere) is on the order of days or (at most) weeks – not thousands of years as this low settling velocity suggests. What happens to these aerosol particles to remove them from the atmosphere?