Homework 5, PHYS 415 (Fluid Mechanics)
Spring 2019
Due Thursday 7th February 2019 at Beginning of Class

As always, turn in your legible and annotated work on separate paper.

1. Consider a two-dimensional steady flow with the flow velocity having the following components:

$$v_x = \alpha y$$
$$v_y = \beta x$$

(α and β are constants).

a) Are there (nonzero) values of α and β that could make this an incompressible flow? If so – what conditions would have to be met? If not, why not?

b) What is the vorticity of this flow?

c) Find a mathematical expression for the streamlines of this flow (assuming $\alpha =\neq 0$ and $\beta \neq 0$).

d) Sketch the streamlines of this flow when $\alpha = \beta > 0$.

e) Find a mathematical expression for the pathlines of this flow (assuming $\alpha \neq 0$ and $\beta \neq 0$). Leave your answer in terms of expressions for $x(t)$ and $y(t)$ given some initial $x_\circ$ and $y_\circ$. (Hint – you have coupled differential equations to deal with here. At least α and β can be treated as constants.)

f) Note that the claim in the problem statement was that this corresponds to a steady flow, yet you (hopefully) found a time-dependent expression in part (e). Does this mean that the streak lines would be blurry? Why or why not?

2. Using index notation (and without appealing to any vector identities), show that another expression for the acceleration for a fluid particle (in an Eularian frame) can be written:

$$\vec{a} = \frac{\partial \vec{v}}{\partial t} + \frac{1}{2}\vec{\nabla}\left(\vec{v}\cdot\vec{v}\right) + \vec{\omega}\times\vec{v}$$

3. Show that the vorticity field for ANY flow satisfies $\vec{\nabla} \cdot \vec{\omega} = 0$. The math here is simple. Make sure you include words and explanations so I can follow the underlying argument.

4. For each of the following velocity fields, determine (i) whether the velocity field is commensurate with an incompressible flow, (ii) the vorticity of the velocity field, (iii) t_{ij}, (iv) d_{ij}, and (v) ω_{ij}. In each field, treat any Greek variables as positive constants with the necessary units.

   a) $\vec{v} = \alpha \hat{x}$
   b) $\vec{v} = \beta \hat{x} + \delta y \hat{y}$
   c) $\vec{v} = \kappa x y z \hat{z}$
   d) $\vec{v} = \eta y z \hat{x} + \zeta x z \hat{y} + \xi x y \hat{z}$