Assignment VI, PHYS 308
Fall 2014
Due 10/10/14 at start of class

NOTE: Just like last homework, please leave your answers in terms of actual numbers (with appropriate units) when possible. Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't read it (or I can't follow your reasoning). Extensive exposition on your thought process or strategy is always appreciated.

1. Assume the temperature at ground level is 300K. Also assume that we're talking about diffusion in air with $\mu = 1.7 \times 10^{-5}$ kg/m/s.

   a) Calculate the diffusion constant D for a 100 μm aerosol particle. (You may assume $C_C = 1$).

   b) How long would it take a collection of 1 μm aerosol particles to diffuse from the center to the edges of a basketball in still air at 300 K? (In other words, how long until the RMS distance traveled by each aerosol would match the radius of a basketball?)

   c) What would C_C be for a 50 nm diameter aerosol? Use $C_C = 1 + \text{Kn}(1.257 + \text{e}^{-1.1/\text{Kn}})$ with $\ell = 8 \times 10^{-7}$ m.

   d) What is the diffusion constant D for the 50 nm aerosol particle in part (c)?

   e) How long would it take a collection of 50 nm aerosol particles to diffuse from the center to the edges of a basketball in still air at 300 K? (In other words, how long until the RMS distance traveled by each aerosol would match the radius of a basketball?)

   f) What is the ratio:

$$\frac{\text{rms distance diffused by 50 nm aerosol particles in 37.5 days}}{\text{rms distance diffused by 1 } \mu\text{m aerosol particles in 37.5 days}}$$

2. Consider the function $n(x,t) = n_\circ + \frac{\Delta N}{(4\pi Dt)^{1/2}} \exp[-x^2/(4Dt)]$.

   a) We will verify that $n(x,t)$ above is a valid solution to the differential equation $\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$. First, calculate $\frac{\partial n}{\partial t}$

   b) Now calculate $\frac{\partial n}{\partial x}$

   c) Now calculate $\frac{\partial^2 n}{\partial x^2}$

   d) Now multiply $\frac{\partial^2 n}{\partial x^2}$ by D and show you get something equivalent to $\frac{\partial n}{\partial t}$.

   e) Consider the integral $\int_{-\infty}^{\infty} [n(x,t) - n_\circ]\mathrm{d}x$. Evaluate it. (You may have to look up the "error function").

   f) What is the value of the integral $\int_{-X}^{X} [n(x,t) - n_\circ]\mathrm{d}x$? (You may leave your answer in terms of error functions and or complementary error functions.)