**Assignment VI, PHYS 230 (Introduction to Modern Physics)**
**Fall 2015**
**Due Thursday, 10/8/15 at start of class**

1. A photoelectric experiment is carried out and, for a particular value of $\lambda$, the stopping potential is $-V_\circ$. Show that the maximum velocity that an electron could take if the battery was set to $+V_\circ$ would be:
$$v = \left(\frac{4e|V_\circ|}{m_e}\right)^{1/2}$$
(ignoring relativistic effects), where $m_e$ is the mass of the electron and $e$ is the magnitude of the charge of an electron.

2. The Rayleigh-Jeans result for the blackbody energy density can be written as:
$$u_T(\lambda)\mathrm{d}\lambda = \frac{8\pi k}{\lambda^5}\lambda T\mathrm{d}\lambda$$
After introducing the notion of quantized energy, the "correct" (Planck) solution is
$$u_T(\lambda)\mathrm{d}\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{\exp(hc/\lambda kT)-1}\right)\mathrm{d}\lambda$$
Take the limit of the Planck solution as $T \rightarrow \infty$ and show whether or not these formally agree in the hypothetical limit of infinite temperature. (Hint – if $T$ is very large, then $hc/\lambda kT$ is small – and it can be treated as a small parameter. You may also want to recall the series expansion of the exponential function).

3. I have a ping-pong ball in my office. It has a diameter of 40 mm (regulation size because I'm a hard-core table tennis player). Let's pretend that I painted it black and it is, for our purposes, a perfect blackbody.

   a) Assume my office is kept at a constant temperature of 293K. How much energy does the ball emit in a year? (You may assume the ball stays in thermal equilibrium at all times).
   b) What would the radius of another blackbody kept at liquid nitrogen temperatures (77K) need to be in order to emit the same amount of power as the ping pong ball?
   c) What is the peak wavelength of the blackbody emission spectrum from the ping pong ball?

4. Quite possibly the most famous problem when starting to study blackbody radiation is to estimate the blackbody temperature of the Earth. Let's walk you through this calculation.

   a) Assume the sun is a perfect blackbody at a temperature of, say, 5800K. What is the total energy outputted by the sun per unit time? (Hint – the Stefan-Boltzmann law will give you the amount of power radiated by the sun per unit area. You may also have to look up the radius of the sun).

   b) Assuming a spherical earth of radius $R_E$ a distance $R_{SE}$ from the sun, what fraction of solar radiation hits the Earth's surface? (Hint – what does the Earth look like if you're standing on the sun? Does it look 2 dimensional or 3? What shape would the Earth look like?) [I'm looking for a symbolic answer to this part of the problem; don't plug in values for $R_E$ and $R_{SE}$ (yet)].

   c) Based on your answer to the above two questions, how much total power is hitting the Earth's surface at all times? You may have to look up values for $R_E$ and/or $R_{SE}$.

   d) If we now treat the Earth as a perfect blackbody in equilibrium, this means that the Earth has to radiate away exactly the amount of power you calculated in the above step. (Power in equals power out for an object in thermal equilibrium). Note that, however, the Earth radiates away energy over its whole surface (not just the surface facing the sun). Using the Stefan-Boltmann law, calculate the equilibrium blackbody temperature of the Earth.

   e) In reality, we've ignored the fact that the Earth is not a perfect blackbody. If we set the albedo of the Earth (the fraction of incoming solar radiation not absorbed by the earth) to be a more realistic value of 0.35 (instead of 0 for a perfect blackbody), what would the equilibrium temperature of the Earth be then? (The power "coming in" is now $(1-A)P_{\text{hitting earth}}$(You may assume the Earth still emits energy like a normal blackbody; it just doesn't absorb it all).

   f) Your answer to part (d) was actually a lot closer to the actual average temperature of Earth's surface than your answer to part (e), despite the fact that your description of the Earth was much more accurate in part (e). This is because we're ignoring one more important thing that causes the Earth to warm. What's the big missing part of the picture so far? (Hint – what does Dr. Larsen research?)

### Mathematica-Based Portion of the Assignment

5. The Planck Energy density is $u_T(\lambda) = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{\exp(hc/\lambda kT)-1}\right)$. The Rayleigh-Jeans relationship is $u_T(\lambda) = \frac{8\pi k}{\lambda^5}\lambda T$. Plot both of these simultaneously on a graph for $T = 3000\text{K}$. (You probably want to use a "LogLinearPlot"). Save a notebook that does this under the name:
`Yourlastname_Phys230_hw6_part1.nb`
and email it to me at `LarsenML@cofc.edu`.

6. Make a manipulate-able plot of the Planck Energy density spectrum (on a "LogLinearPlot") where the user can change and/or animate $T$ over a range of 77K (the temperature of liquid nitrogen) to 6000K (about the blackbody-equivalent temperature of our sun). If you can, try to make it so that the y-axis stays fixed as the animation proceeds. (Before messing with it, it will constantly be resizing automatically which makes the change in the magnitude of the plot harder to detect). Save a notebook that does this under the name:
`Yourlastname_Phys230_hw6_part2.nb`
and email it to me at `LarsenML@cofc.edu`.

7. The maximum blackbody emission occurs when the slope of $u_T(\lambda) = 0$. It turns out that we can't actually use pencil and paper to find this in terms of elementary functions because we get something called a transcendental equation. This problem is meant to walk you through how to deal with a situation like this.

   a) First, take the Planck expression for $u_T(\lambda)$ and differentiate with respect to $\lambda$. You'll have to recall the chain rule and some other stuff. (Be careful with your algebra). (Do *not* do this part in Mathematica – though if you want to check your work with it, that's ok.)

   b) After setting this derivative equal to zero and doing some simplification, you should come up with an expression similar to the following:

   $$0 = \text{stuff}\,(\text{different stuff} - n)$$

   with $n$ a prime number. If you divide both sides by "stuff" and make the substitution $x = hc/\lambda kT$, you should be able to rewrite the above expression as:

   $$n = \frac{x\exp(x)}{\exp(x) - 1}$$

   This is what we call a transcendental equation, and our usual mathematical toolkits aren't particularly helpful here. Enter Mathematica. There are a number of ways to solve this. I'm going to outline 2 in particular and I'd like you to try both.

      i) Let us rewrite the above expression as follows:

      $$f(x) = \frac{x\exp(x)}{\exp(x) - 1} - n$$

      (remember, from part (a) you know what $n$ is supposed to be). One way of finding $x$ is to plot $f(x)$ and find the intersection with the $x$ axis (a.k.a. the $x$-intercept). We want a pretty precise result here (at least 4 or 5 digits past the decimal point), so you will have to keep narrowing the possible range of $x$ down to smaller and smaller regions until you have a reliable value for where it crosses the $x$ axis. (As a hint, it should be near, but slightly less than, $n$).

ii) You can get Mathematica to try and solve this for you through numerical approximation. Find out how to write a function in Mathematica. (If you're stuck, feel free to ask classmates or myself). Define the function $f(x)$ as described above, and then use the "Solve" command to find the value of $x$ needed so that $f(x) = 0$. Note that at first, it isn't likely to give you anything particularly useful because this is a transcendental relationship. You'll want a numerical approximation. Add the appropriate "numerically approximate this" modifier and get a numerical approximation of $x$ to 10 digits past the decimal.

c) Using your answers to part (b) (which hopefully are the same for both methods), note that $x = \frac{hc}{\lambda kT}$. Normally, Wien's law is written in the form $\lambda T =$ constant. Using the value of $x$ you found, find the constant. (It should be very close to the accepted value). (Remember to include units). (Do not do this portion of the problem in Mathematica).

**What to turn in on this problem**

I know this was kind of long and elaborate and some parts of this involved Mathematica and some parts involved computation. For part (a), turn in (as part of the rest of your assignment) your manipulations to get a final expression of the type $0 = \frac{d\rho}{d\lambda}$ =stuff(different stuff - $n$). With "stuff" and "different stuff" being combinations of constants and "$n$" being a prime number. For part (b), turn in a saved mathematica notebook (via email to me at `LarsenML@cofc.edu`) that clearly shows (i) a plot, and (ii) a numerical solution of the resulting transcendental equation to find $x$. Name the notebook `Yourlatname_Phys230_hw6_part3.nb`. For part (c), turn in (with the rest of your non-Mathematica homework) a manipulation of $hc/\lambda kT = x$ with the now known value of $x$ to find the value of the Wien's law constant. Try to keep this organized as best you can so I can follow everything you've done.