Assignment VI, PHYS 230 (Modern Physics)
Fall 2019 Due Thursday October 31st, 2019 at Start of Class

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

1. Estimate the de Broglie wavelengths of the following:

   a) An electron in the ground-state Bohr orbit.

   b) A baseball pitched at 100 miles per hour.

   c) The Earth. (You may assume the sun is stationary. You may have to look up some numbers).

   d) An average Sodium molecule in an ideal gas at 500 pK. (The average speed of a molecule in an ideal gas can be computed via $v_{\text{avg}} = \left(\frac{8kT}{\pi m}\right)^{1/2}$ with k the Boltzmann constant k. A picoKelvin is 10^{-12}K.) (Think carefully about what mass to use).

   e) Compare your answer in part (d) to the accepted value of Sodium's atomic radius (approximately 186 picometers).

2. Let a particle of mass m be constrained to be between points $-a/2$ and $+a/2$ on the x-axis.

   a) What is the minimum uncertainty in the particle's momentum?

   b) What is the minimum uncertainty in the particle's kinetic energy? (You may ignore relativistic effects and assume there is no uncertainty in the particle's mass).

   c) Using your result from (b) above, calculate the minimum energy of an electron between $-a/2$ and $a/2$ when $a \sim 5.3 \times 10^{-11}$ m. (This distance is known as the "Bohr radius" and corresponds to the most likely distance between the proton and electron in a Hydrogen atom in its ground state).

   d) Using your result from (b) above, calculate the minimum energy of an electron confined between $-a/2$ and $a/2$ when $a = 0.01$ m.

   e) Using your result from (b) above, calculate the minimum energy of a 100 mg bead moving on a thin (frictionless) wire between two rigid stops that are 2 cm apart.

3. In order to locate a particle to within 5×10^{-12} meters using light, the wavelength of the light must be at most 5×10^{-12} meters.

   a) Calculate the energy of a photon with $\lambda = 5 \times 10^{-12}$ m.

   b) Calculate the momentum of a photon with $\lambda = 5 \times 10^{-12}$ m.

   c) If this light bounces off an electron leaving an uncertainty $\Delta x = 5 \times 10^{-12}$ m to its position, what is the minimum uncertainty in the electron's momentum?

4. An excited state of a certain nucleus has a half-life of 2.3 ns.

   a) Taking this to be the uncertainty Δt for emission of a photon, calculate the minimum uncertainty in the frequency of the emitted light.

   b) If the emitted light is expected to have a wavelength 0.05nm, what is $\Delta f/f$ for this light? ($\Delta f/f$ can be interpreted as the fractional uncertainty of the frequency).

5. A sound wave is generated from a tone-generator at 880 Hz. How long would you have to play this sound so that the uncertainty in the frequency of the generated sound is at most 1 Hz?

6. In class (a while ago), we found the ground state radius of the Bohr Hydrogen atom by using a semi-classical approach. Using a somewhat similar semi-classical approach, you can find $a_\circ$ using only (1) the Heisenberg Uncertainty principle, and (2) semi-classical force balance ($F_e = F_c$). Find $a_\circ$ from these two ideas. (Your answer, if done correctly, will differ from the Bohr value of $a_\circ$ by a dimensionless constant. Don't worry about that – we're looking for you to get the right functional dependence on things like m_e, $\epsilon_\circ$, etc.)

7. Use MATLAB to make a professional-quality log-log plot of the minimum value of ΔE as a function of Δt according to the Heisenberg Uncertainty Principle. Make sure you have both macroscopic and subatomic relevant scales on the plot (e.g. have values of t ranging from the orbital time of a ground-state Hydrogen electron in the Bohr theory up to reasonable scales for humans – e.g. an hour or so. As usual, send me an email to LarsenML@gmail.com with an m-file that I can run to generate the figure for myself. Make sure your axes are labelled with units on your axes.