Assignment VI, PHYS 301 (Classical Mechanics)
Spring 2015
Due 2/27/15 at start of class

1. An undamped oscillator reaches maximal displacement from equilibrium $x_\circ$ and has maximal velocity $v_{\max}$. What is the period of oscillation in terms of $x_\circ$ and $v_{\max}$?

2. Look at the figure below, which is a schematic picture of a "weeble". (A weeble is a child's toy, with the commercial tagline "they wobble, but they don't fall down"). They are solid objects with a hemispherical base of radius R, and another shape above it. The center of mass is some distance d above the bottom of the weeble. (You can assume that d is a known quantity). Let the weeble have total mass m.

   a) Write a potential energy function $U(\theta)$ for the weeble in terms of m, d, R, g, and θ where θ is the angular rotation of the weeble from the upright position. (You may assume $|\theta| \leq \pi/2$ so that the contact point between the weeble and the floor is still on the hemisphere).

   b) There is only one equilibrium angle for this system. It should be easy to find (if nothing else, just from symmetry). The behavior at the equilibrium angle can be either stable or unstable. Find a relationship between d and R that constrains the equilibrium to be a stable one. Interpret this physically.

   c) Find the angular frequency of small oscillations about the stable equilibrium if $d = \frac{R}{3}$. (Note – make sure your answer has the right units! Things might fall apart more naturally in polar coordinates, but θ^2 is not a distance!)

3. A mass m oscillates on a spring with spring constant k. The amplitude of the oscillation is A. At some moment (which we'll call $t = 0$, the mass is at position $x = A/2$ and is moving to the right; at $t = 0$ the mass collides and sticks to a second mass m. The speed of the resulting mass $2m$ right after the collision is half the speed of the moving mass m right before the collision (due to conservation of momentum).

   a) What is the amplitude of this new oscillation?

   b) Write an expression for $x(t)$ that is valid for all $t > 0$.

4. A particle of mass m is subject to a spring force $-kx$ and also a driving force $f_\circ m\cos(\omega t)$; however there is no damping force.

    a) After rewriting this system, you end up with $\ddot{x} + \omega_\circ^2 x = f_\circ \cos(\omega t)$. Find a particular solution to this differential equation by guessing $x(t) = C\cos(\omega t) + D\sin(\omega t)$. Your answer should be in terms of $\omega_\circ$, ω, and $f_\circ$ only.

    b) Based on your answer to part (a), you should be able to come up with the general solution to $x(t)$ pretty easily. From this solution, consider the following: If this system had initial conditions $x(t=0) = \frac{3f_\circ}{(\omega_\circ^2-\omega^2)}$ and $v(t=0) = 0$, what is the acceleration of this system be at $t = 0$?

5. In class, I repeatedly claimed (and showed) that the oscillations of any system about a stable equilibrium should behave like a simple harmonic oscillator. (Or, as I put it in class, "everything in nature near a stable equilibrium acts like a spring"). Let's put this to the test for a particular system. A particle of mass m is subject to the following potential energy function:

$$U(x) = -Ax^3 e^{-\alpha x}$$

    You may assume A and α are positive real constants, and x is constrained between 0 and $+\infty$.

    a) Find $x_\circ$ (the x coordinate associated with the stable equilibrium).

    b) Find $U(x_\circ)$.

    c) Near $x_\circ$, you should be able to write $U(x) \approx U(x_\circ) + \frac{1}{2}k(x - x_\circ)^2$. What is the angular frequency of small oscillations ($\omega_\circ$) for this system near the equilibrium in terms of A, α, and m?

6. A mass on the end of a spring is released from rest at horizontal extension beyond equilibrium A. The moving mass achieves maximal speed v_1. The experiment is repeated, but now the entire system is immersed in a fluid that causes the motion to be critically damped. In this case, the mass achieves maximal speed v_2. Find the ratio v_1/v_2.

7. You probably remember the system below from the last homework. Just as a refresher...A solid triangular wedge is placed on Earth. The hypotenuse (of length D) has a surface that is frictionless. To the top of the wedge is affixed a (massless) spring with spring constant k and equilibrium length $x_\circ$. To the bottom of the wedge, a massless spring with spring constant $3k$ and equilibrium length $x_\circ$ is placed. A point mass m is attached to both springs. $D = 2x_\circ$ so that, if this system were placed in space (in the absence of gravity), the system would be in a stable equilibrium with the point mass exactly halfway down the hypotenuse. The angle of inclination of this wedge is ϕ as shown. Let the variable ℓ indicate the distance of the mass from the bottom right corner of the triangle.

    In the last homework, you showed that the equilibrium position of this system is at $\ell = x_\circ - \frac{mg\sin\phi}{4k}$. Your task is to find the angular frequency of small oscillations with respect to this equilibrium. (Assume no damping).