Assignment VI, PHYS 308
Fall 2016
Due 10/14/16 at start of class

NOTE: Just like last homework, please leave your answers in terms of actual numbers (with appropriate units) when possible. Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't read it (or I can't follow your reasoning). Extensive exposition on your thought process or strategy is always appreciated.

1. Approximate the Reynolds number for each of the following (and, for each, determine if the flow is likely laminar or turbulent). In some cases, you may need to look up some values and do some extra calculations to determine things.

   a) You walking at a regular pace in still air outdoors.

   b) A baseball being pitched towards home plate.

   c) An olympic swimmer swimming in a pool.

   d) A 2 mm steel ball-bearing falling at its terminal fall-velocity in a large vat of honey. (Use $\mu_h = 2$ Pa s, $\rho_h = 1420$ kg/m^3, and $\rho_s = 8050$ kg/m^3 for the viscosity of the honey and the densities of the honey and steel, respectively).

2. Air flows through a 0.25 inch diameter tube at a rate of 30 liters per minute. Is the flow turbulent? (We will say that the transition between laminar and turbulent flow for pipe flow occurs when the Reynolds number hits about 3000). If necessary, assume the ambient temperature is 20 degrees Celcius and the ambient pressure is 1 atm. [This isn't a completely pointless question. In my lab, we have some optical particle counters used for sampling aerosols. These detectors move approximately 30 liters per minute through an opening that is close to 0.25 inches in diameter].

3. To calibrate a rain-sensing device that we have (a 2-dimensional video disdrometer), we have a process by which we drop spherical steel ball-bearings through the device from a known height. I'm not 100% sure the exact height, but – for the sake of this problem – let's say that it is 60 cm above the sensing surface. Let's say we are dropping 10 mm spheres made out of solid steel (density of 8050 kg/m^3), initially at rest (hence "dropping") from a height of 60 cm through air (near the surface of the Earth). For these spheres, we have historically ignored both wind and drag. Let's make sure this is reasonable.

   a) Assuming $C_D = 0.44$, what is the terminal velocity (in air) of these spheres?

   b) If you neglect air resistance entirely, the downward velocity of a falling object is $v = v_\circ + gt$ and, in particular, if dropped, you have $v = gt$. Use the formula derived in class to find the numerical ratio $v(t)/(gt)$ for these spheres for (i) $t = 0.1$ second, (ii) $t = 0.5$ seconds, (iii) $t = 1.0$ seconds, (iv) $t = 2.0$ seconds, (v) $t = 5.0$ seconds, and (vi) $t = 10$ seconds. (I personally used MATLAB to help me out here. Saved me a bunch of computation. You may solve this however you'd like.)

   c) Based on your answers to (a) and (b) (or other information, if necessary), explain why we don't have to factor in drag for these spheres.

4. In class, I developed a differential equation and solved it for a settling aerosol particle subject to Stokes' drag. A simplified version of this equation (valid when $\rho_f \ll \rho_p$) can be written:

$$\tau \frac{\mathrm{d}v}{\mathrm{d}t} = \tau g - v$$

with τ still taking the same definition used in class. The solution to this differential equation, when the particle starts falling from rest (e.g $v(t=0)=0$) can be written:

$$v(t) = \tau g \left(1 - e^{-t/\tau}\right)$$

a) Verify (by substituting $t = 0$) that this gives $v(t=0) = 0$.

b) Verify (by substituting the proposed solution into the differential equation) that this proposed solution satisfies the differential equation.

c) Use the solution to determine the acceleration $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ as a function of time.

d) Find $v(t \to \infty) = v_t$.

e) Let $0 \leq \alpha \leq 1$. How long (in terms of τ) would it take for an aerosol falling at rest to reach a fall speed of αv_t?

f) Given your answer to part (e) above, how long would it take for a 100 nm diameter aerosol particle (with density 1300 kg/m^3) to reach 95% of its terminal fall-speed? (Use $Kn = 1$ for the 100 nm aerosol particle falling in air).

g) How far would the aerosol particle fallen in part (f) in the time it takes to reach 95% of its terminal fallspeed? (Your answer is very likely to surprise you. Hint – the distance fallen after time t can be calculated from $\Delta z = \int_0^t v(t)\mathrm{d}t$).

5. (Extra Credit!) Use your favorite computer algebra system/coding language/computational resource to draw a log-log plot of F_{drag} as a function of particle size. Have curves for Stokes' drag (dotted line) and the empirically adjusted value (solid line) as presented in class. You may assume that the fluid is air (near the surface of the Earth), the relative velocity is 10 m/s, and have the x-axis (particle size) range from 0.1 nm to 1 cm. Note that this is 8 orders of magnitude difference in size, so you don't want to have a step size of 0.1 nm unless you have a week of computer time to kill. (Ask in class for hints!) In addition to the graph, please turn in your code/mathematica session/maple desktop/matlab code/excel spreadsheet/etc. (Feel free to assume $C_C = 1$ throughout the entire range of sizes, even though that's a bit of an oversimplification).

6. (More Extra Credit!) I have uploaded some real data from the Aerodynamic Particle Sizer Spectrometer that I took at a field-study in New Mexico in January of 2007 at `http://larsenml.people.cofc.edu/sample_data.csv`. This file (which you should be able to open with pretty much any text editor) is organized so that each row of the file corresponds to a 1 second measurement. The first row of the file gives the maximum diameter (in micrometers) of detected particles in each size bin. Different size bins are separated by commas. You should have a total of 5052 rows of data (the first row being the bin-sizes, and the remaining 5051 rows indicating observed aerosol counts in each size bin for each measurement). Assume all detected particles have a diameter equal to the maximum diameter in their respective bins.

   a) What is the mean diameter for all detected particles in the event?

   b) Make a plot of the mean diameter as a function of time for the event. This will mean that each row ultimately ends up giving you a separate value of $\bar{D}_p$. Plot this as a function of elapsed time.

   c) Much more commonly used than mean diameter in atmospheric science is the mass-weighted mean diameter. If the "standard" mean diameter of N particles is given by:

   $$\bar{D}_p = \frac{1}{N}\sum_{i=1}^{N} D_i$$

   then the "mass-weighted mean diameter" is given by:

   $$\bar{D}_m = \frac{\sum_{i=1}^{N} D_i^4}{\sum_{i=1}^{N} D_i^3}$$

   Make a plot similar to the one you created in part (b), except do so for the mass-weighted mean diameter. Note that it should be substantially different than your answer to part (b).

   d) Make a histogram of the mean diameters for each measurement in the event. (Each second has its own mean diameter. Use these to create a histogram of mean diameters for the entire event).

   e) Make a histogram of the mass-weighted mean diameters for each measurement in the event.

   f) Based on your answers for (a-e), comment on what you have seen/what this teaches you.