Homework 6, PHYS 415 (Fluid Mechanics)
Spring 2019
Due Thursday 14th February 2019 at Beginning of Class

As always, turn in your legible and annotated work on separate paper.

1. An air-filled iron spherical shell (density of iron $\rho = 7860\ \text{kg/m}^3$) has an apparent mass of 324 kg in air and 247 kg when submerged in liquid water. What are the inner and outer radii of the shell? (For this problem, you may NOT neglect the buoyant force on the air-filled iron sphere when it is in air.)

2. All parts of this problem assume an incompressible stationary fluid having mass density ρ_f. For this problem, you may also neglect the buoyant force due to displacing air above the fluid layer.

   a) Within the fluid floats an object of volume V_{obj} with uniform density ρ_{obj}. How much fluid volume is displaced by the object? You may assume the average density of the object is less than ρ_f.

   b) Another object floats in the fluid; a thin rod of length L and cross section A having a mass density $\rho(x) = \rho_\circ + kx^3$ (x is understood to run from 0 to L.) If the rod *just barely* floats, what is the length of the rod in terms of k, ρ_f, and $\rho_\circ$?

3. Glycerine has a density of ρ_g and liquid water has density ρ_w; you may assume both fluids are incompressible. A container has a deep layer of glycerine below a deep layer of water. A uniform cube with density $\rho_w < \rho < \rho_g$ with length-side H and aligned with one of its faces parallel to the water-glycerine interface is gently placed near the boundary between the fluids.

   a) Eventually, the object will come to rest at its equilibrium point. What is the depth of the cube's intrusion into the glycerine layer when it is at its equilibrium position?

   b) Before arriving at that equilibrium position, the cube will bob up and down in oscillatory motion. What is the angular frequency of small oscillations of the object about its equilibrium point? (You may neglect viscous damping). You may also ignore any influence the moving object may have on the fluids; just pretend that they can't be mixed (the container is very large and the oscillations are very small).

4. In class, we have taken some pains to make sense of the constitutive relationship between stress and strain within a fluid. For this problem, I want you to dive into some textbook (or, probably more likely, a google-hole) to learn a little bit about a non-fluids-based constitutive relationship. I have assigned a different constitutive relationship to each of you – below this problem, find your assigned relationship. For your system/relationship, I want you to clearly describe the following:

   a) What physical quantities are related to each other in your constitutive relationship? What type of quantities are they (scalars, vectors, rank 2 or more tensors, etc.)

   b) What material quantity (or quantities) are involved that make this a constitutive relationship.

   c) Is the material quantity truly a scalar, or is it possibly a tensor? (Justify your answer).

   d) Describe a context in which knowledge of the material quantity in this constitutive relationship could be practically useful.

   e) How could you design an experiment to measure the material quantity in question? (Go into enough detail to convince me that – with a large enough budget – you could actually do it. Don't just say something like "measure the voltage" unless it is obvious how that could be done).

   Aidan: Piezooptic Effect
   Gavin: Hall Effect
   Grant: Fick's Law of Diffusion
   Jason: Fourier's Law of Thermal Conduction
   Christian: Pyroelectric Effect
   Bridget: Electrical Conductivity
   Vincent: Specific Heat Capacity
   Andrew: Stefan-Boltzmann Blackbody Law
   Thomas: Peltier Effect
   Max: Ohm's Law (not just $V = IR$)
   Payden: Piezoelectric Effect
   Quinn: Seebeck Effect