Assignment VII, PHYS 308
Fall 2014
Due 10/17/14 at start of class

NOTE: Please leave your answers in terms of actual numbers (with appropriate units) when possible. Please provide full, legible, easy to follow solutions to the following problems. I can't give you credit if I can't follow your reasoning. Extensive exposition on your thought process or strategy is always appreciated.

1. I gave you (in class) the following expression for the Diffusion Constant of an aerosol particle: $D = \frac{kTC_C}{3\pi\mu D_p}$. (Note that this expression has units of m^2/s). Another route to the diffusion equation developed from kinetic theory gives:

$$D^* = \frac{\lambda^2}{2t_{\text{ave}}}$$

where λ is the mean free path between collisions for a particle, and t_{ave} is the mean time between collisions. In practice, both of these expressions are legitimate ways of writing the diffusion constant. The first method makes a bit more sense for aerosols (where there is a particle and a fluid medium that are clearly different from each other). Using D^* makes more sense if you're just talking about diffusion of a single gas into, say, a vacuum or something. This problem is about playing with the two different definitions of D simultaneously.

a) For methane, $D = 1.78 \times 10^{-5}$ m^2/s. Assume $\lambda = 90$ nm. Find the mean time between intermolecular collisions for methane.

b) What is the mean *instantaneous* speed for a methane molecule? (Hint....speed = distance/time).

c) What is the root mean squared displacement from the origin for a methane molecule released at $t = 0$ at the origin and left to travel for 3 minutes?

d) What is the actual distance that the methane molecule moved in this time? (Not the displacement, but the distance).

e) Derive an expression for (root mean squared displacement from the origin)/(distance traveled) as a function of t. Leave your answer in terms of D, t, and v_{ave} (the average instantaneous velocity of a methane molecule).

f) It is physically impossible for the diffusion distance to be larger than the distance traveled. This means that the expression you calculated in part (e) only can hold when $t > t_\circ$. Find the smallest possible value of $t_\circ$ for methane.

2. Under certain circumstances, the radiation inside the sun can be treated like a gas of photons with the mean free path between interactions nominally 1 mm. (We say interactions instead of collisions because we're really talking about absoprtion and re-emission here. Instead of "colliding" with other photons, a photon moves for nominally 1 mm before being absorbed by something. Then it is re-emitted. You may assume this absorption/re-emission process is instantaneous). (This is a rather crude model, but it works for some situations).

   a) Under this basic model, how long (on average) would it take a photon created at the center of the sun to diffuse out to the surface of the sun? (It is not, of course, the "same" photon – but let's say we're tracing the energy path instead of the photon's path. We're abusing language a bit, but the language is at least evocative). Leave your answer in years.

   b) What is the ratio:
   (root mean squared displacement from the center of the sun)/(distance traveled)
   just as the photon gets to the sun's surface?

3. In class, we developed the following expression for G_{rel}:

$$G_{\text{rel}} = 4\pi r^2 \sigma - \frac{4}{3}\pi r^3 nkT \ln\left(\frac{e}{e_s}\right)$$

   a) Find the value of r when G_{rel} reaches a maximum. (We call this value r_c for the critical radius). You may assume all parameters in the problem except for r can be treated as constants. You may also assume $e > e_s$.

   b) Find the value of G_{rel} when $r = r_c$. Please simplify your answer as much as you reasonably can.

   c) For $e > e_s$, $G_{\text{rel}} = 0$ at two values of r. The first is at $r = 0$. Find the other value of r that makes this true. We'll refer to the value of r you find here as $r_{\text{2nd zero}}$ (in the following parts of this problem).

   d) Let's compare $r_{\text{2nd zero}}$ to r_c. You might expect $r_{\text{2nd zero}} = 2r_c$ if the curve is completely symmetric for positive values of G_{rel}. Is $r_{\text{2nd zero}}$ greater than, equal to, or less than $2r_c$?

   e) How many liquid water molecules would be in a sphere of radius $r_{\text{2nd zero}}$? Assume $e/e_s = 1.042$, $\sigma = 0.076$ J/m^2, and $T = 270$K.

   f) Technically, anything with $r > r_{\text{2nd zero}}$ has $G_{\text{rel}} < 0$ and thus is energetically favorable to exist compared to vapor. However, we do not see homogeneous nucleation of water drops that start at $r_{\text{2nd zero}}$ and grow. Explain why we don't see this.