Assignment VII, PHYS 230 (Modern Physics)
Fall 2019 Due Thursday November 7th, 2019 at Start of Class

As always, turn your legible and complete answers in on separate paper. Remember, I can't give partial credit unless I can follow what you've done. Including words is usually a good thing for you.

1. Two waves travel simultaneously along a long wire. The equations defining the waves are:

$$u_1(x,t) = (0.002\text{ m})\cos\left(\left[8.0\ \frac{1}{\text{m}}\right]x - \left[400\frac{1}{\text{s}}\right]t\right)$$
$$u_2(x,t) = (0.002\text{ m})\cos\left(\left[7.6\ \frac{1}{\text{m}}\right]x - \left[380\frac{1}{\text{s}}\right]t\right)$$

   where u and x are in meters and t is in seconds.

   a) Add these two waves together to form a single equation for $u_1(x,t) + u_2(x,t)$. Leave your answer in the general form $U(x,t) = A\text{trig}(\text{mess}_1)\text{trig}(\text{mess}_2)$ with each "trig" indicating a sine or a cosine.

   b) What is the phase velocity of the resultant wave?

   c) What is the group velocity of the resultant wave?

   d) Calculate the spatial interval Δx between successive zeros of the group (packet) and relate it to Δk.

2. Show that, in general, $v_g = v_p + k\frac{\mathrm{d}v_p}{\mathrm{d}k}$.

3. Consider a dispersion relationship for a string: $\omega^2 = \frac{T}{\mu}k^2 + \alpha k^4$ with T the tension on the string, μ the linear mass density of the string (mass per unit length), and α a positive constant.

   a) What are the units of α?

   b) What is the phase velocity of the string?

   c) What is the group velocity of the string? (Simplify your answer! No fractions within fractions!)

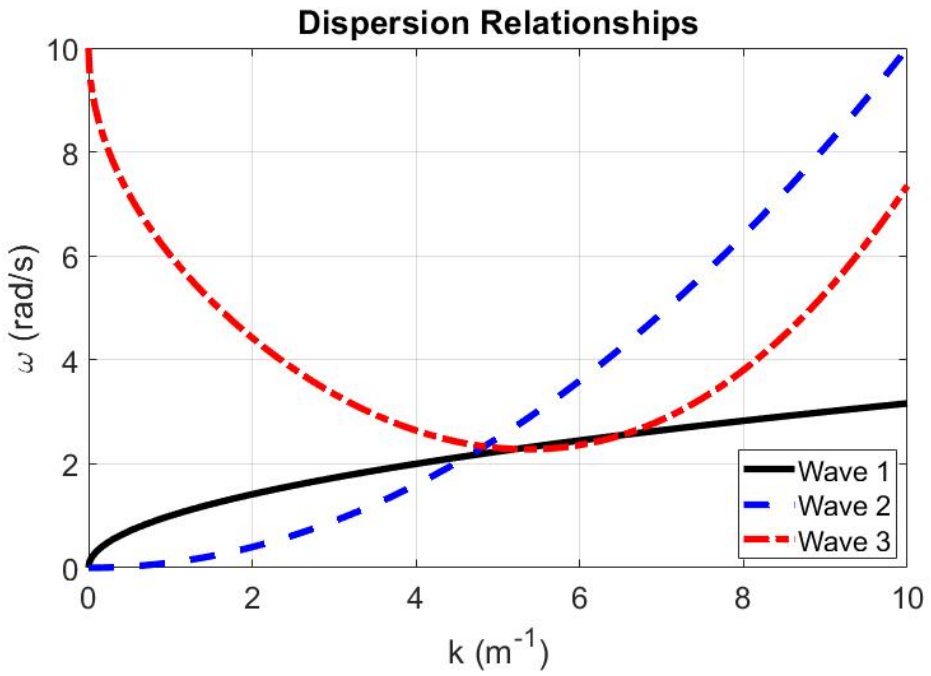
4. Recall that, for light in a vacuum, $c = \lambda f$.

   a) Rewrite this relationship in terms of k and ω.

   b) Based on your answer to part (a), what is the phase velocity for light in a vacuum?

   c) Based on your answer to part (a), what is the group velocity for light in a vacuum?

5. Examine the plot of dispersion relationships below, and answer the following questions.



a) Except for when $k = 0$, are there any places for any of these waves where the phase or the group velocity is zero? If so, name the wave (1,2, or 3) where it happens, the wave number it happens at, and whether it is the phase or group velocity that vanishes.

b) Except for when $k = 0$, are there any places for any of these waves where the phase or group velocity is negative? If so, name the wave (1, 2, or 3), the wave number(s) it happens at, and whether it is the phase or the group velocity that is negative.

c) At $k = 2$ m^{-1}, is the phase or group velocity larger for wave 1?

d) At $k = 6$ m^{-1}, is the phase or group velocity larger for wave 2?

e) At $k = 8$ m^{-1}, is the phase or group velocity larger for wave 3?

6. Let us state that the probability density function describing the position of a particle is given by the relationship:

$$p(x) = \begin{cases} A(b - |x|)^3 & |x| \leq b \\ 0 & \text{otherwise} \end{cases}$$

Given that the particle must be *somewhere* we know that $\int_{-\infty}^{\infty} p(x)\mathrm{d}x = 1$. From this information, determine what the value of A must be. (You may have a b in your answer).