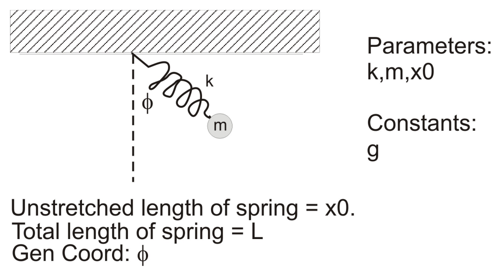
Assignment VII, PHYS 301 (Classical Mechanics)
Spring 2017
Due 3/31/17 at start of class

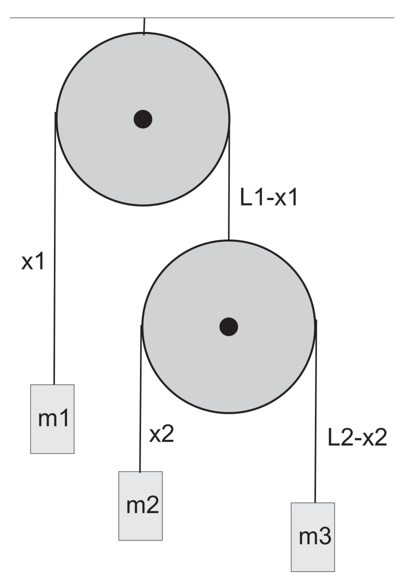
1. Consider the "spring pendulum" shown below. A massless spring with equilibrium length $x_\circ$ and spring constant k hangs from a level ceiling. Use the principles of Lagrangian mechanics to develop (coupled) differential equations for $\phi(t)$ and $L(t)$ (the generalized coordinates we'll use to describe this system). You may keep parameters $x_\circ$, m, and g in your answer.



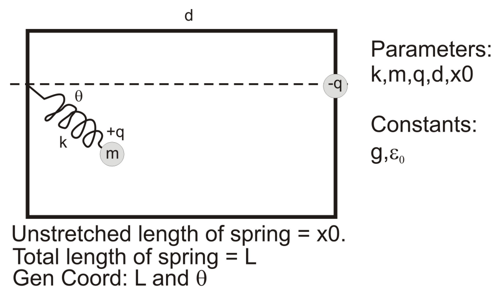
2. Consider the "double Atwood's" machine shown below. This system has two degrees of freedom and we will use generalized coordinates x_1 and x_2 as shown. L_1, $L_)2$, m_1, m_2, and m_3 are constants that may appear in your solutions. Assume the pulleys are massless and that we're on Earth with g downward on the figure.

   a) What is the Lagrangian for this system? (Leave your answer in terms of m_1, m_2, m_3, x_1, x_2, L_1, L_2, g, $\dot{x}_1$, and $\dot{x}_2$ only.)

   b) Use the Lagrangian to find (coupled) differential equations for $\ddot{x}_1$ and $\ddot{x}_2$.

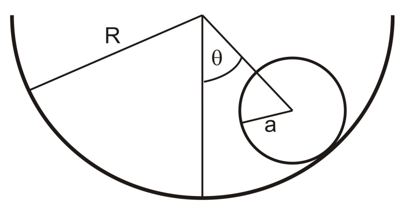


3. A massless spring of unstretched length $x_\circ$ is connected to a wall. At the end of this spring is a mass m that also carries total charge $+q$. Directly across from the wall-attachment point of the spring (a distance d away) is another point charge $-q$. This is a system with two degrees of freedom; we'll use L (the current length of the spring) and θ (the current angle the spring makes with respect to the horizontal) as these two degrees of freedom. Construct the Lagrangian for this system and use it to derive (coupled) differential equations of motion that determine $L(t)$ and $\theta(t)$.



4. A uniform sphere of radius a and mass M is constrained to roll without slipping (don't forget the rotational energy!) on the lower half of the inner surface of a hollow cylinder of inside radius R. You may assume the cylinder remains stationary with respect to Earth. (In particular, the cylinder is nailed to the ground and does not roll).

   a) Determine the Lagrangian function.
   b) Write down the equation of motion. (You do not need to solve the differential equation; just get the differential equation).
   c) What is the frequency (NOT angular frequency) of small oscillations?



5. Assume the Earth is a uniform sphere with density ρ.

   a) Show that a particle dropped into a straight hole (of negligible width) passing through Earth's center would execute simple harmonic motion.
   b) Find the period of the oscillation of the particle described in part (a). [Your answer should only depend on Earth's density, ρ, not its size!]
   c) Recall (or look up, if you have to) the Earth's radius and mass. Use this information to numerically compute what the frequency of oscillation of the particle in parts (a) and (b) would have to be. (You may use a calculator).

6. A small hole of negligible diameter is bored along a radius into a sphere whose density ρ at a given point is a power-law function of r, the distance of the point from the center of the sphere. (In other words, you may assume that for $r < R$ the density is described by $\rho(r) = Ar^a$ for some real constants A and a). It is found that the gravitational force exerted on the particle in the hole is independent of the distance of the particle from the center of the hole. If the total mass of the sphere is M and its total radius is R, what is $\rho(r)$?