Homework 7, PHYS 415 (Fluid Mechanics)
Spring 2019
Due Thursday 21st February 2019 at Beginning of Class
Last Homework Before Midterm!

As always, turn in your legible and annotated work on separate paper.

1. Assume the Earth is Spherical, and ignore the effects of the Centrifugal and Euler forces on falling objects as well as the effect of air resistance. A stone is dropped from a 200-m high bridge at our latitude (32.78°). How far (and in which cardinal direction) does the stone land from the point immediately below the release point of the stone on the Earth's surface?

2. Take a 250-metric-ton locomotive at our latitude rolling on a straight track at 60 m/s.

   a) What lateral force is exerted on the tracks?

   b) What angle should the train track be banked to account for this force?

3. In lecture, we have conducted a fairly in-depth scale analysis of synoptic flows in the atmosphere. This problem works on extending those results. As we did in class, let $f \equiv 2\Omega \sin\varphi$.

   a) Start with the equations $\frac{\mathrm{D}v_x}{\mathrm{D}t} = fv_y$ and $\frac{\mathrm{D}v_y}{\mathrm{D}t} = -fv_x$. Use these equations to determine the period of inertial oscillations. (Hint: take some derivatives and look for a diffeq that reminds you of oscillations).

   b) Use your result from part (a) above to make a plot of T (in days) as a function of φ with Earth's actual Ω. Use some sort of computer plotting program to make this plot, and consider using a semilog axis for your y-coordinate.

MORE ON BACK

4. We haven't talked about it too much yet, but arguably the most important dimensionless number to characterize a flow is the Reynolds number: $\mathrm{Re} = \frac{\rho L v}{\mu}$. When Reynolds numbers are small, flows tend to be "laminar" – there isn't much lateral mixing in the fluid flows tend to be more regular or predictable than in other contexts. At larger Reynolds numbers, things get turbulent and complicated. It is worthwhile calculating a few of these Reynolds numbers for different flow scenarios to get a sense of if inertial effects dominate (for turbulent flows) or viscous effects dominate (for laminar flows). For each of the following, compute the Reynolds number.

   a) A person walking through a field outdoors at a normal walking pace.

   b) A steel ball bearing (let's say diameter = 2 cm and density = 8050 kg/m^3) dropped in very large vat of honey and allowed to reach its terminal velocity. Assume that the drag force on the ball bearing is equal to $3\pi\mu D v$ where v is the ball-honey relative velocity. For honey, use the data on this webpage: http://tinyurl.com/larsen-honey and assume a temperature of 20°C.

   c) Water coming out of a garden hose at 20°C at a flow rate of 1.00 cfm (cubic feet per minute).

5. Even in scenarios where the Reynolds number is large, viscosity still plays an important role in a (potentially thin) layer close to the boundary of the interface between the fluid and object, because velocity gradients in that layer can still be significant enough to augment the effect of viscosity there. The thickness of this boundary layer is $L/\sqrt{\mathrm{Re}}$. For each of the scenarios described above, determine the thickness of the associated boundary layer.