Assignment VIII, PHYS 230 (Introduction to Modern Physics)
Fall 2015
Due Thursday, 10/22/15 at start of class

1. Numerically approximate the de Broglie wavelengths of the following:

   a) An electron with kinetic energy 13.6 eV.

   b) The fastest human being ever recorded to run (instantaneous velocity). (Google some stuff if necessary).

   c) The Earth. (You may assume the sun is stationary. Again, you may have to Google some stuff).

   d) An average Sodium molecule in an ideal gas at 500 pK. (The average speed of a molecule in an ideal gas can be computed via $v_{\text{avg}} = \left(\frac{8kT}{\pi m}\right)^{1/2}$ with k the Boltzmann constant. A picoKelvin is 10^{-12} K). (Think carefully about what mass to use).

   e) Compare your answer in part (d) to the accepted value of Sodium's atomic radius (approximately 186 picometers). Briefly comment.

2. The formulation of much of the wave mechanics that will appear in the coming weeks relies on the formulation of a "wave equation". The most familiar wave equation is a differential equation of the form:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

   where v is the velocity of the wave, and $u(x,t)$ is a function describing the thing that's waving. Although many people in this class have not had differential equations – so "solving" this isn't necessarily fair for me to ask....there *are* things we can do with this equation that might help us understanding what's going on a bit better.

   a) Verify (by explicitly taking the necessary derivatives) that $u = A\cos(kx - \omega t)$ is a solution to this equation. (In doing this, you will find a relationship that must be obeyed between k, ω, and v for this to be a valid solution. What is that relationship?) Treat A, k, and ω as real constants.

   b) Verify (by explicitly taking the necessary derivatives) that $u = Be^{i(kx-\omega t)}$ is a solution to this equation as well. (Again, you'll find the same relationship between k, ω, and v.) Treat B, k, and ω as real constants.

(More on back)

3. Two waves travel simultaneously along a long wire. Their wave functions (the solutions to the wave equation above) are:

$$u_1(x,t) = (0.002 \text{ m}) \cos\left(\left[8.0 \ \frac{1}{\text{m}}\right] x - \left[400\frac{1}{\text{s}}\right] t\right)$$
$$u_2(x,t) = (0.002 \text{ m}) \cos\left(\left[7.6 \ \frac{1}{\text{m}}\right] x - \left[380\frac{1}{\text{s}}\right] t\right)$$

where u and x are in meters and t is in seconds.

a) Add these two waves together to form a single equation for $u_1(x,t) + u_2(x,t)$. Leave your answer in the general form $U(x,t) = 2A\text{trig}(\text{mess}_1)\text{trig}(\text{mess}_2)$ with each "trig" indicating a sine or a cosine (sort of similar to equation 4.19 in your text, but with the messes involving time as well).

b) What is the phase velocity of the resultant wave?

c) What is the group velocity of the resultant wave?

d) Calculate the spatial interval Δx between successive zeros of the group (packet) and relate it to Δk.

4. Let a particle of mass m be constrained to be between points $-a/2$ and $+a/2$ on the x-axis.

a) What is the minimum uncertainty in the particle's momentum?

b) What is the minimum uncertainty in the particle's kinetic energy? (You may ignore relativistic effects and assume there is no uncertainty in the particle's mass).

c) Using your result from (b) above, calculate the minimum energy of an electron between $-a/2$ and $a/2$ when $a \sim 5.3 \ \times 10^{-11}$ m. (This distance is known as the "Bohr radius" and corresponds to the most likely distance between the proton and electron in a Hydrogen atom in its ground state).

d) Using your result from (b) above, calculate the minimum energy of an electron confined between $-a/2$ and $a/2$ when $a = 0.01$ m.

e) Using your result from (b) above, calculate the minimum energy of a 100 mg bead moving on a thin (frictionless) wire between two rigid stops that are 2 cm apart.